Let $f$ be a real analytic function in a neighborhood of the origin such that $f(0) = 0, \nabla f(0) = 0$. Then there are constant $C > 0$ and $\beta \left( \frac{1}{2} \leq \beta < 1 \right)$ such that for $x$ near origin, we have

$$|\nabla f(x)| \geq C|f(x)|^\beta$$

This is so called the Lojasiewicz gradient inequality for real analytic functions. A simple question is: How sharp is the inequality in any sense? By Integrating over the inequality we have

$$\int_{\{f\neq 0\}} |\nabla f(x)| \frac{dx}{f(x)} \geq C \int_{\{f\neq 0\}} \frac{1}{|f|^{(1-\beta)d}}.$$ 

In this talk, we will prove that the left hand side integral is always infinite, and on the other hand, the right hand side integral could be finite in general using a recent geometric proof Lojasiewicz inequalities by P. Feehan. It is this phenomenon that we conclude the Lojasiewicz gradient inequality is not necessarily sharp in $L^d$ sense.

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