A Thurston map is a branched covering map on a topological 2-sphere for which the forward orbit of each critical point under iteration is finite. Each such map gives rise to a fractal geometry on its underlying 2-sphere. The study of these maps and their associated fractal structures links diverse areas of mathematics such as dynamical systems, classical conformal analysis, hyperbolic geometry, Teichmüller theory, and analysis on metric spaces. In my talk I will report on some recent developments and open problems in this area. (Received July 04, 2019)