In general, the logarithmic capacity of compact sets in the plane is not continuous under Hausdorff convergence: for example, a sequence of finite sets $E_n$ can converge to the unit disk $D$ in the Hausdorff metric, where $\text{cap} E_n = 0$ but $\text{cap} D = 1$. In 2006, Aseev and Lazareva proved that the capacity is continuous if the sets $E_n$ are uniformly uniformly perfect, that is there exists $\alpha > 0$ such that each $E_n$ is $\alpha$-uniformly perfect. 

We consider a sequence of compact sets $E_n$ where each set $E_n$ is uniformly perfect with some constant $\alpha_n$, but $\alpha_n$ may approach 0 as $n \to \infty$. Such sequences naturally arise during the construction of non-self-similar fractal sets. The main result is that if the Hausdorff distance $d_H(E_n, E)$ decays sufficiently fast, then $\text{cap} E_n \to \text{cap} E$. The required rate of convergence of $d_H(E_n, E)$ to 0 depends on the rate of convergence of $\alpha_n$ to 0. (Received July 08, 2019)