A fundamental problem in discrete geometry asks to pack lines (one-dimensional subspaces) in real or complex space without forming sharp angles. By definition, an optimal packing is one in which the sharpest angle between any two lines is made large as possible, given the number of lines and the ambient dimension. Among the known optimal packings, many display extraordinary symmetry. This talk focuses on a special case of this phenomenon: doubly transitive lines. Such lines are not only equiangular, but also optimally packed in projective space. Moreover, for every sequence of doubly transitive lines there exists a choice of unit-norm line representatives whose Gram matrix carries a very special type of association scheme. By leveraging this fact, we provide a partial classification of doubly transitive lines, namely those with almost simple symmetries. (Received July 15, 2019)