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David A Cardon* (`cardon@mathematics.byu.edu`), Department of Mathematics, TMCB 275, Brigham Young University, Provo, UT 84602. *Interlacing properties of coefficient polynomials in differential operator representations of real-root preserving linear transformations.*

We study linear transformations $T: \mathbb{R}[x] \rightarrow \mathbb{R}[x]$ of the form $T[x^n] = P_n(x)$ where $\{P_n(x)\}$ is a real orthogonal polynomial system. In particular, we are interested in linear transformations that map polynomials with all real zeros to polynomials with all real zeros. It is well known that any transformation $T: \mathbb{C}[x] \rightarrow \mathbb{C}[x]$ has a differential operator representation $T = \sum_{k=0}^{\infty} \frac{Q_k(x)}{k!} D^k$. We seek to understand the behavior of the transformation T by studying the roots of the $Q_k(x)$. We prove three main things. First, we show that the only case where the $Q_k(x)$ are constant and $\{P_n(x)\}$ are an orthogonal system is that when the orthogonal system is a shifted set of generalized probabilist Hermite polynomials. Second, we show that the coefficient polynomials $Q_k(x)$ have real roots when the $P_n(x)$ are the physicist Hermite polynomials or the Laguerre polynomials. Lastly, we show that in these cases, the roots of successive polynomials strictly interlace, a property that has not yet been studied for coefficient polynomials. We conclude by presenting several open problems. (Received February 27, 2020)