

1155-11-171

Michael J. Mossinghoff* (mossinghoff@alumni.stanford.edu), Center for Communications Research, 805 Bunn Dr., Princeton, NJ 08540, and **Timothy S. Trudgian** and **Tomás Oliveira e Silva**. *The Distribution of k -free Numbers*.

For an integer $k \geq 2$, let $Q_k(x)$ denote the number of k -free integers in $[1, x]$, that is, integers that have no divisor of the form p^k with p prime. Since the k -free integers have density $1/\zeta(k)$ in \mathbb{Z}^+ , it is natural to consider the error incurred when estimating $Q_k(x)$ using this density, so let $R_k(x) = Q_k(x) - x/\zeta(k)$. It is well known that $R_k(x) = \Omega(x^{1/2k})$ and $O(x^{1/k})$, and often conjectured that $R_k(x) = O(x^{\frac{1}{2k} + \epsilon})$. While the Riemann hypothesis (and more) would follow if in fact $R_k(x) = O(x^{1/2k})$, we discuss why this bound is often thought to be false, despite experimental investigations that might seem to support it. We also describe some improved lower bounds on $|R_k(x)/x^{1/2k}|$ for various k , deduced by establishing weak linear independence of ordinates of certain zeros of the Riemann zeta function. In addition, we report on some statistics for square-free and cube-free integers obtained by computations up to 10^{18} , including some results on gaps between such numbers. (Received January 11, 2020)