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The Bilinear Hilbert Transform is a prototypical modulation invariant multi-linear singular operator.

$$\text{BHT}_\alpha(f_1, f_2)(x) := \int_{\mathbb{R}} \int_{\mathbb{R}} \widehat{f}_1(\xi_1) \widehat{f}_2(\xi_2) 1_{[0, +\infty)}(\xi_1 - \alpha \xi_2) e^{2\pi i(\xi_1 + \xi_2)x} d\xi_1 d\xi_2$$

It arises in many contexts including Cauchy integrals along Lipschitz curves and in the study of Calderón commutators relevant for semi-linear non-constant-coefficient elliptic PDEs.. The bounds

$$\|\text{BHT}_\alpha(f_1, f_2)(x)\|_{L^{p'_3}(\mathbb{R})} \leq C_{p_1, p_2, \alpha} \|f_1\|_{L^{p_1}(\mathbb{R})} \|f_2\|_{L^{p_2}(\mathbb{R})}$$

hold for any $p_{1,2,3} \in (1, +\infty)$ with $\sum_{i=1}^3 p_i^{-1} = 1$.

A longstanding open problem is whether the above bounds holds with a uniform constant in the parameter α . The dyadic analog of this problem has been solved in Oberlin, Thiele '10.

We will give a general overview of the topic. We will then sketch how the framework of outer measure L^p spaces in time-scale-frequency space plays a crucial role in proving the result and how the problem gives useful insights into embeddings of functions into times-scale-frequency space. (Received January 14, 2020)