

1155-42-323

Rui Han, Vjekoslav Kovac, Michael Lacey, Jose Madrid* (jmadrid@math.ucla.edu) and
Fan Yang. *Improving estimates for discrete polynomial averages.*

For a polynomial P mapping the integers into the integers, define an averaging operator $A_N f(x) := \frac{1}{N} \sum_{k=1}^N f(x + P(k))$ acting on functions on the integers. We prove sufficient conditions for the ℓ^p -improving inequality

$$\|A_N f\|_{\ell^q(\mathbb{Z})} \lesssim_{P,p,q} N^{-d(\frac{1}{p}-\frac{1}{q})} \|f\|_{\ell^p(\mathbb{Z})}, \quad N \in \mathbb{N},$$

where $1 \leq p \leq q \leq \infty$. For a range of quadratic polynomials, the inequalities established are sharp, up to the boundary of the allowed pairs of (p, q) . For degree three and higher, the inequalities are close to being sharp. In the quadratic case, we appeal to discrete fractional integrals as studied by Stein and Wainger. In the higher degree case, we appeal to the Vinogradov Mean Value Theorem, recently established by Bourgain, Demeter, and Guth. (Received January 17, 2020)