

1156-20-163

Ignat Soroko (ignat.soroko@gmail.com). *Ranks of joins and intersections in free groups.*

The famous Hanna Neumann Conjecture (now the Friedman–Mineyev Theorem) stipulates that for the ranks of arbitrary subgroups H and K of a nonabelian free group we have:

$$\text{rank } H \cap K - 1 \leq (\text{rank } H - 1)(\text{rank } K - 1).$$

It is an interesting open question to quantify this bound with respect to the rank of $H \vee K$, the subgroup generated by H and K . We describe a set of realizable values $(\text{rank } H \vee K, \text{rank } H \cap K)$ for arbitrary H, K , and conjecture that this locus is complete. Using the approach developed by Dicks in the context of his Amalgamated Graph Conjecture, we prove that the locus is complete when $\text{rank } H = 2$ and show that the region

$$\text{rank } H \vee K \geq \text{rank } H + \text{rank } K - 3 \quad \& \quad \text{rank } H \cap K \geq 4$$

consists of non-realizable values, thus resolving the remaining open case of R. Guzman’s conjecture in the affirmative and obtaining applications to 3-dimensional topology. (Received January 28, 2020)