

Let C be a nonempty subset of an arbitrary set X and $f : X \rightarrow \mathbb{R}$. The objective is to minimize $f(x)$ over $x \in C$. We get x^k , for $k = 1, 2, \dots$, by minimizing $G_k(x) = f(x) + g_k(x)$ over all $x \in X$. We call this approach an *auxiliary-function* (AF) method if $g_k : X \rightarrow [0, +\infty]$, $g_k(x^{k-1}) = 0$, and $g_k(x) < +\infty$ if and only if $x \in C$. Then $\{f(x^k)\} \downarrow \beta^* \geq -\infty$. We consider conditions on the auxiliary functions g_k that guarantee that $\beta^* = \beta \doteq \inf_{x \in C} f(x)$.

An AF algorithm is said to be in the SUMMA class if the SUMMA Inequality, $G_k(x) - G_k(x^k) \geq g_{k+1}(x)$, for all $x \in X$, holds for all k , in which case it follows that $\beta^* = \beta$. We consider a variety of AF algorithms that either are in the SUMMA class or can be reformulated to be such. We also study some AF algorithms that are not in the SUMMA class, but for which $\beta^* = \beta$. This leads to a larger class, the SUMMA2 class of AF algorithms.

An AF algorithm is a proximal minimization algorithm (PMA) if $g_k(x) = d(x, x^{k-1})$, where $d : X \times X \rightarrow [0, +\infty]$ is a distance, so that $d(x, y) = 0$ if and only if $x = y$. Optimization transfer (OT) algorithms in statistics can be reformulated as PMA. (Received December 26, 2019)