We derive an explicit formulation for fractional powers of the Lamé-Navier operator of linear elasticity. We show that this “fractional Lamé-Navier” operator appears in several models in continuum mechanics. The operator coincides with the equations of motion in state-based peridynamics for a particular choice of parameters. The half-power of the Lamé-Navier operator also appears as the Dirichlet-to-Neumann map associated to the constitutive equations of an isotropic homogeneous elastic solid occupying a half-space. We establish basic calculus properties of the fractional operators, derive fundamental solutions, and establish solvability results in classes of well-known function spaces. Finally, we find an explicit Caffarelli-Silvestre extension problem associated to the fractional Lamé-Navier equation. We show that mean-value properties, regularity, and calculus identities for solutions to the fractional Lamé-Navier equations can be derived from this degenerate elliptic system. (Received August 18, 2021)