The adjunction inequality is a key tool for bounding the genus of smoothly embedded surfaces in 4-manifolds. Various forms of this inequality have been established using gauge theoretic methods such as Donaldson, Seiberg-Witten and Heegaard-Floer. However, these generally require global geometry, such as a symplectic structure or basic classes. In this talk, we will introduce polyhedral decompositions of almost-complex 4-manifolds. These can be used to give adjunction criterion in much larger classes of smooth 4-manifolds. (Received August 09, 2021)