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Abdul Basit* (basit.abdul@gmail.com), **Artem Chernikov**, **Sergei Starchenko**, **Terence Tao** and **Chieu-Minh Tran**. *Zarankiewicz's problem for semilinear hypergraphs.*

A bipartite graph $H = (V_1, V_2; E)$ with $|V_1| + |V_2| = n$ is *semilinear* if $V_i \subseteq \mathbb{R}^{d_i}$ for some d_i and the edge relation E consists of the pairs of points $(x_1, x_2) \in V_1 \times V_2$ satisfying a fixed Boolean combination of q linear equalities and inequalities in $d_1 + d_2$ variables for some q . We show that for a fixed k , the number of edges in a $K_{k,k}$ -free semilinear H is almost linear in n , namely $|E| = O_{q,k}(n^{1+\varepsilon})$ for any $\varepsilon > 0$; and more generally, $|E| = O_{q,k,r}(n^{r-1+\varepsilon})$ for a $K_{k,\dots,k}$ -free semilinear r -partite r -uniform hypergraph.

As an application, we obtain the following incidence bound: given n points and m open boxes with axis parallel sides in \mathbb{R}^d such that their incidence graph is $K_{k,k}$ -free, there can be at most $O_k((n + m)^{1+\varepsilon})$ incidences. Instead of boxes one can take polytopes cut out by the translates of an arbitrary fixed finite set of halfspaces. (Received September 01, 2020)