Markov chain Monte Carlo (MCMC) approaches are traditionally used for uncertainty quantification in inverse problems where the forward problem is described by a partial differential equation (PDE). However, the use of MCMC algorithms is prohibitively expensive in certain applications where each log-likelihood evaluation may require hundreds to thousands of PDE solves. We show how to mitigate this computational cost by using several randomized techniques and discuss various ways in which the randomized approximations can be embedded within MCMC algorithms. We demonstrate the accuracy and computational benefits of our proposed algorithms on a model application from diffuse optical tomography. (Received August 13, 2020)