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**Gareth A. Jones\*** ([g.a.jones@maths.soton.ac.uk](mailto:g.a.jones@maths.soton.ac.uk)), 15 Nile Road, Highfield,  
SOUTHAMPTON, NH SO17 1PF, United Kingdom, and **Alexander K. Zvonkin.** *Klein, dessins  
d'enfants and projective primes.* Preliminary report.

Motivated by Klein's paper of 1879 on equations of degree 11, we studied dessins of type  $(3, 2, p)$  and degree  $p$  for primes  $p$ . This involved studying the perfect transitive permutation groups of prime degree, all 'known' as a result of the classification of finite simple groups: they include the projective groups  $\mathrm{PSL}_n(q)$  where their natural degree  $(q^n - 1)/(q - 1)$  is a prime  $p$ . It is unknown whether there are finitely or infinitely many such 'projective primes'  $p$ . Examples include the Fermat primes for  $n = 2$  and the Mersenne primes for  $q = 2$ . When  $n = 3$ , where we found more than  $1.29 \times 10^8$  primes  $p = 1 + q + q^2 \leq 10^{22}$ , an application of the Bateman–Horn Conjecture suggests that there are infinitely many projective primes, and allows us to make estimates for their density agreeing with computer searches in this range to within 0.35%. For  $n = 3$  each prime  $p$  yields  $(p - 1)/3e$  dessins of type  $(3, 2, p)$ , degree  $p$ , genus  $(q - 3)(q + 1)/12$  and monodromy group  $\mathrm{PSL}_3(q)$  where  $q$  is the  $e$ th power of an odd prime; there is a similar result when  $q = 2^e$ . (Received February 15, 2021)