Classical ramification theory deals with extensions of complete discrete valuation rings with perfect residue fields. The study of arbitrary valuation rings with possibly imperfect residue fields and possibly non-discrete valuations of rank $\geq 1$ presents many fascinating complications. In particular, the defect could be non-trivial when in positive residue characteristic (i.e. we can have a non-trivial extension, such that there is no extension of the residue field or the value group).

In this talk, we will present some results for Artin-Schreier extensions of arbitrary valuation fields in positive characteristic $p$. These results relate the “higher ramification ideal” of the extension with the ideal generated by the inverses of Artin-Schreier generators via the norm map. This ideal plays the role of the classical Swan conductor in the arbitrary case. We will also introduce a generalization and further refinement of Kato’s refined Swan conductor in this setting. Similar results are true in mixed characteristic $(0, p)$. If time permits, we will briefly discuss the connection between these results and some recent work (joint with K. Kato) on upper ramification groups. (Received August 21, 2021)