1172-11-16 **Qi Han** (qhan@tamusa.edu), Texas A&M University-San Antonio, San Antonio, TX 78224, and
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Algebraic differential independence regarding the Riemann $\boldsymbol{\zeta}$-function and the Euler $\boldsymbol{\Gamma}$-function.

It is a profound result of Hölder in 1887 saying that the Euler gamma-function $\boldsymbol{\Gamma}$ cannot satisfy any nontrivial algebraic differential equation having polynomial coefficients in $\mathbf{C}$. David Hilbert, in his lecture addressed to the International Congress of Mathematicians at Paris in 1900 for his famous 23 problems, stated in Problem 18 that the Riemann zeta-function $\boldsymbol{\zeta}$ cannot satisfy any nontrivial algebraic differential equation having polynomial coefficients in $\mathbf{C}$, either; this problem was generalized around the 1930's by Mordukhai-Boltovskoi and Ostrowski independently, and further extended by Voronin.

In 2007, Lawrence Markus conjectured that $\boldsymbol{\Gamma}$ and $\boldsymbol{\zeta}$ are mutually algebraically differential independent. In this talk, we study this question and show a positive partial answer to it; that is, we prove $\boldsymbol{\zeta}$ can not be a solution to any nontrivial algebraic differential equation whose coefficients are polynomials in $\boldsymbol{\Gamma}, \boldsymbol{\Gamma}^{(n)}$ and $\boldsymbol{\Gamma}^{(\ell n)}$ over the ring of polynomials in $\mathbf{C}$, where $\ell, n \geq 1$ are positive integers. (Received July 26, 2021)