1172-28-159 **Kornelia Hera***, herak@uchicago.edu, and **Tamas Keleti** and **Andras Mathe**. *Fubini-type theorems for Hausdorff dimension and their connection to unions of lines.*

It is well known that for Hausdorff dimension the naive Fubini theorem does not hold. Namely, there exist sets $E \subset \mathbb{R}^n$ such that for all $x \in \mathbb{R}$, the vertical sections $E_x = \{y \in \mathbb{R}^{n-1} : (x, y) \in E\}$ have Hausdorff dimension s, and $\dim_H E > s+1$.

We prove a weaker variant of the Fubini theorem for Hausdorff dimension. Namely, for any Borel set B there is a small subset $G \subset B$ (in an appropriate sense) such that for $B \setminus G$ the naive Fubini theorem holds.

Our results combined with the problem of how large a union of lines must be depending on the Hausdorff dimension of the family of lines constituting the union imply that in fact for small unions of lines, we do not even have to remove a subset for the naive Fubini theorem to be valid. Our results are closely connected to the Kakeya problem.

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