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**Gennady Uraltsev\*** (gennady.uraltsev@gmail.com), 302 Kerchof Hall – Math Department,  
141 Cabell Drive, P.O. Box 400137, Charlottesville, VA 22904. *Banach space-valued time frequency  
scale analysis.*

Singular Integral Operator theory and, Calderón-Zygmund theory specifically, provides a vast array of tools for dealing with operators similar to the Hilbert transform

$$Hf(x) := \int_{\mathbb{R}} f(x-y) \frac{dy}{y},$$

ubiquitous in Complex Analysis, semi-linear PDEs, etc. Results for  $C$ -valued functions were extended to Banach space-valued functions thanks to Burkholder-Bourgain's groundbreaking work on the deep relation between Banach space geometry and boundedness properties of vector-valued SIOs.

$C$ -valued bounds for multilinear SIOs, like the bilinear Hilbert transform

$$BHT[f_1, f_2](x) = \int_{\mathbb{R}} f_1(x-t) f_2(x+t) \frac{dt}{t},$$

are classic in time-frequency-scale analysis but Banach-space valued results have appeared only recently. Connections with Banach space geometry from linear theory are just starting to be investigated.

We present a sharp result in terms of interpolation exponents for  $BHT$  and consider some natural generalizations under stronger assumptions on the Banach spaces in play.

Open questions and generalizations to non-commutative analysis abound and would come hand-in-hand with progress in understanding SIOs with worse singularities than of Calderón-Zygmund type that can often be realized as SIO-valued CZ operators. (Received August 24, 2021)