Decidability and periodicity of translational tilings.

Let $G$ be a finitely generated abelian group, and $F_1, \ldots, F_J$ be finite subsets of $G$. We say that $F_1, \ldots, F_J$ tile $G$ by translations, if $G$ can be covered by translated copies of $F_1, \ldots, F_J$, without any overlaps.

Given some finite sets $F_1, \ldots, F_J$ in $G$, can we decide whether they admit a tiling of $G$? Suppose that they do tile $G$, do they admit a periodic tiling? A well known argument of Hao Wang (’61), shows that these two questions are closely related. In the talk, we will discuss this relation, and present some results, old and new, about the decidability and periodicity of translational tilings, in the case of a single tile ($J = 1$) as well as in the case of a multi-tileset ($J > 1$). (Received August 28, 2021)