We present a data-driven framework for exterior calculus on manifolds. This framework is based on representations of vector fields, differential forms, and operators acting on these objects in frames (overcomplete bases) for $L^2$ and higher-order Sobolev spaces built entirely from the eigenvalues and eigenfunctions of the Laplacian of functions. Using this approach, we represent vector fields either as linear combinations of frame elements, or as operators on functions via matrices. In addition, we construct a Galerkin approximation scheme for the eigenvalue problem for the Laplace-de-Rham operator on 1-forms, and establish its spectral convergence. We present applications of this scheme to a variety of examples involving data sampled on smooth manifolds and the Lorenz 63 fractal attractor. (Received August 31, 2021)