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**Jonathan W. Siegel\*** (jus1949@psu.edu), 722 W Whitehall Rd, State College, PA 16801. *The Approximation Theory of Shallow Neural Networks.*

A shallow neural network is a linear combination of ridge functions whose profile is determined by a fixed activation function. We will introduce spaces of functions which can be efficiently approximated by shallow neural networks for a wide variety of activation functions and analyze their properties. Specifically, we will compute their metric entropy and  $n$ -widths, which are fundamental quantities in approximation theory that control the limits of linear and non-linear approximation and statistical estimation for a given class of functions. Consequences of these results include: the optimal approximation rates which can be attained for shallow neural networks, that shallow neural networks dramatically outperform linear methods of approximation, and indeed that shallow neural networks outperform all stable methods of approximation on these spaces. This will provide insights into how neural networks break the curse of dimensionality. Finally, we discuss algorithmic aspects of approximation by shallow networks. Specifically, we analyze a class of greedy algorithms and show that they can attain the theoretically optimal approximation rates. (Received September 01, 2021)