We study the problem of finding a local optimum and a Brouwer fixed point of a function in the black box model, where there is an upper bound $k$ on the number of rounds of interaction with the function oracle. Rounds model distributed settings, where each query takes resources to complete and is executed on a separate processor.

We focus on the $d$-dimensional grid $[n]^d$, where the dimension $d$ is a constant. For local search, when the number of rounds $k$ is constant, the query complexity is $\Theta\left(n^{d^k/n} \cdot k\right)$ for both deterministic and randomized algorithms. When the number of rounds is polynomial, i.e. $k = n^\alpha$ for $0 < \alpha < d/2$, the query complexity is at most $O\left(n^{(d-1)/d^2} \cdot n^\alpha\right)$ and at least $\tilde{\Omega}\left(\max(n^{(d-1)/\alpha}, n^{d/2})\right)$ for randomized algorithms.

These bounds also imply a characterization of the query complexity of computing an $\epsilon$-approximate Brouwer fixed-point in the $d$-dimensional unit cube $[0,1]^d$ in $k$ rounds, where we find the query complexity is $\Theta\left((1/\epsilon)^{d^k/n} \cdot k\right)$ for both deterministic and randomized algorithms. (Received September 01, 2021)