1164-37-95 **Victor LeBlanc*** (vleblanc@uottawa.ca), Department of Mathematics and Statistics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada. *Degenerate Hopf Bifurcations in DDEs and Endemic Bubbles.*

For $(\lambda, \mu) \in R^2$, let

$$\dot{x}(t) = \mathcal{L}(\lambda, \mu)x_t + \mathcal{F}(x_t, \lambda, \mu) \tag{1}$$

denote a family of nonlinear retarded FDEs, with linear part $\mathcal{L}$ and nonlinear terms $\mathcal{F}$. Suppose that the equilibrium $x = 0$ has a pair of eigenvalues

$$\xi(\lambda, \mu) \pm i\omega(\lambda, \mu), \quad \text{such that} \;\; \xi(0,0) = 0, \;\; \omega(0,0) = \omega_0 > 0$$

and

$$\xi_\lambda(0,0) = 0. \tag{2}$$

The crossing condition of the Hopf bifurcation theorem is violated. We give a classification of degenerate Hopf bifurcation diagrams for (**??**) near $(0,0)$. These results are applied to the SIS model incorporating delayed behavioral response

$$\dot{y}(t) = -y(t) + R_0 h(y(t-\tau), p)y(t)(1 - y(t)) \tag{3}$$

where $y(t)$ represents the proportion of infected individuals, R_0 is the basic reproduction number, and the behavioral function $h : [0,1] \times R \longrightarrow (0,1]$ is such that $h_y(y,p) \leq 0$, $h(0,p) = 1$ and $h(1,p) < 1$. We show that the phenomenon of *endemic bubbles* is a consequence of a degenerate Hopf bifurcation which occurs in the (R_0, p) parameter space from the endemic equilibrium of (**??**). (Received January 14, 2021)