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Song Yu. *Symmetries of Spatial Graphs in 3-manifolds.*

We consider when automorphisms of a graph can be induced by homeomorphisms of embeddings of the graph in a 3-manifold. In particular, we prove that every automorphism of a graph is induced by a homeomorphism of some embedding of the graph in a connected sum of one or more copies of $S^2 \times S^1$, yet there exist automorphisms which are not induced by a homeomorphism of any embedding of the graph in any orientable, closed, connected, irreducible 3-manifold. We also prove that for any 3-connected graph G , if an automorphism σ is induced by a homeomorphism of an embedding of G in an irreducible 3-manifold M , then G can be embedded in an orientable, closed, connected 3-manifold M' such that σ is induced by a finite order homeomorphism of M' , though this is not true for graphs which are not 3-connected. Finally, we show that many symmetry properties of graphs in S^3 hold for graphs in homology spheres, yet we give an example of an automorphism of a graph G that is induced by a homeomorphism of some embedding of G in the Poincaré homology sphere, but is not induced by a homeomorphism of any embedding of G in S^3 . (Received September 15, 2021)