The Connes-Kasparov isomorphism between the ring of representations of a maximal compact subgroup $K$ of a real reductive group $G$ and the K-theory of the reduced C*-algebra of $G$ has long be known to yield a correspondence between discrete series of $G$ and certain representations of $K$. In this talk, I will report on joint work with N. Higson, Y. Song and X. Tang providing a characterization of all the representations of $G$ that contribute to the K-theory of $C^*_r(G)$ in terms of Dirac cohomology and leading to a new, representation-theoretic proof of the Connes-Kasparov isomorphism. (Received January 22, 2022)