1176-52-235 **Timothy F Havel*** (tfhavel@alum.mit.edu). *Heron's Formula for the Tetrahedron, and Some of Its Combinatorial Corollaries.* Preliminary report.

A natural extension of Heron's 2000 year old formula for the area of a triangle to the volume of a tetrahedron is presented. This extension gives the fourth power of the volume as a polynomial in six simple rational functions of the areas of its four faces and of its three medial parallelograms, which will be referred to herein as interior faces. Geometrically, these rational functions are the areas of the triangles into which the exterior faces are divided by the points at which the tetrahedron's in-sphere touches those faces. This leads to a conjecture as to how the formula likely extends to n-dimensional simplices for all $n > 3$. Remarkably, for $n = 3$ the zeros of the overall polynomial constitute a five-dimensional real semi-algebraic variety consisting almost entirely of collinear tetrahedra with some of their vertices at infinity. These unconventional Euclidean configurations can be identified with antipodal pairs of points on the Klein quadric, wherein four-point configurations in the finite affine plane constitute a distinguished three-dimensional subset. The talk will close by noting that the algebraic structure of the zeros in the finite affine plane naturally defines the associated 4-element, rank-3 chirotope, aka affine oriented matroid. (Received January 24, 2022)