Advances in SOVIET MATHEMATICS

Volume 9

Representation Theory and Dynamical Systems

A. M. Vershik Editor



American Mathematical Society

Representation Theory and Dynamical Systems

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Editor's Preface

This volume includes papers presenting recent research (1989–1991) of the participants of the Leningrad Seminar on representation theory, dynamical systems, and their applications.

The seminar was organized by the editor of this volume in the mid-sixties at the Mathematics Department of Leningrad State University. Since 1980 it is also a seminar of the Leningrad Branch of the Steklov Mathematical Institute.

To some extent, our seminar is continuing the research of V. A. Rokhlin's Ergodic Seminar, which he created in 1960 and headed till the end of the sixties. Still, from its very beginning, the main purpose of our seminar was representation theory, the theory of C^* - and W^* -algebras, in their connections both with ergodic theory and with purely algebraic questions. During the early years of the seminar, problems of measure theory in linear spaces, smooth dynamical systems, geometric and analytic problems of representation theory, and Lie group theory were studied; later combinatorics in its contemporary setting and other topics arose. All these problems were attracting mathematicians specializing in different fields.

This is not the first time that the proceedings of the seminar are published. Papers by its participants have been appearing regularly since 1980 in the series Zapiski Nauchnykh Seminarov LOMI. Differential Geometry, Lie Groups and Mechanics (L. D. Faddeev, ed.), volumes 3–10. They were translated in Journal of Soviet Mathematics, vols. 26 (1985), 36 (1987), 38 (1987), 41 (1988), 47 (1989), and later.

The papers included in this collection represent, although not completely, current topics of the seminar. It is a difficult matter to divide contemporary mathematics into separate fields (and there is no need in that). The synthetic nature of the process of creating mathematics can be best observed in the case of mathematical physics and its algebraic aspects, always a central subject of our seminar. Topics such as Lie groups and their representations, infinite-dimensional groups, topology and dynamical systems interact with each other, creating numerous relationships with various mathematical theories. That is why the title of this volume is imprecise. The above-mentioned subjects have dominated seminar meetings during the last three or four years.

EDITOR'S PREFACE

Other topics, not reflected properly in the present volume, but arising constantly at the seminar's workshops, are worth mentioning: combinatorics, asymptotic theory of groups and representations, representations of infinitedimensional groups, random walks, nonholomorphic differential geometry, and applications. We hope these will be included in forthcoming volumes of the seminar's proceedings.

Leningrad, March 1991

A. M. Vershik Translated by A. B. SOSSINSKY

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