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Shinichi Mochizuki

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This book lays the foundation for a theory of uniformization of p-adic hyperbolic curves and their moduli. On the one hand, this theory generalizes the Fuchsian and Bers uniformizations of complex hyperbolic curves and their moduli to nonarchimedian places. For that reason, the theory is referred to as p-adic Teichmüller theory for short. On the other hand, the theory may be regarded as a somewhat precise hyperbolic analog of the Serre-Tate theory of ordinary abelian varieties and their moduli.

This approach to the uniformization of p-adic hyperbolic curves and their moduli was initiated by Mochizuki in a series of earlier papers. In this book, Mochizuki aims to bridge the gap between the approach presented and the classical uniformization of a hyperbolic Riemann surface that is studied in undergraduate complex analysis. Mochizuki's work on inter-universal Teichmüller theory is an extension of the results presented here.

Features:

- Presents a systematic treatment of the moduli space of curves from the point of view of p-adic Galois representations.
- Treats the analog of Serre-Tate theory for hyperbolic curves.
- Develops a p-adic analog of Fuchsian and Bers uniformization theories.
- Gives a systematic treatment of a "nonabelian example" of p-adic Hodge theory.

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