AMS/IP

Studies in Advanced Mathematics

S.-T. Yau, Series Editor

Some Current Topics on Nonlinear Conservation Laws

Lectures at the Morningside Center of Mathematics, 1

Ling Hsiao and Zhouping Xin, Editors

American Mathematical Society • International Press

Selected Titles in This Series

15 **Ling Hsiao and Zhouping Xin, Editors,** Some Current Topics on Nonlinear Conservation Laws, 2000

14 **Jun-ichi Igusa,** An Introduction to the Theory of Local Zeta Functions, 2000

13 **Vasilios Alexiades and George Siopsis, Editors,** Trends in Mathematical Physics, 1999

12 **Sheng Gong,** The Bieberbach Conjecture, 1999

11 **Shinichi Mochizuki,** Foundations of p-adic Teichmüller Theory, 1999

10 **Duong H. Phong, Luc Vinet, and Shing-Tung Yau, Editors,** Mirror Symmetry III, 1999

9 **Shing-Tung Yau, Editor,** Mirror Symmetry I, 1998

8 **Jürgen Jost, Wilfrid Kendall, Umberto Mosco, Michael Röckner, and Karl-Theodor Sturm,** New Directions in Dirichlet Forms, 1998

7 **D. A. Buell and J. T. Teitelbaum, Editors,** Computational Perspectives on Number Theory, 1998

6 **Harold Levine,** Partial Differential Equations, 1997

5 **Qi-keng Lu, Stephen S.-T. Yau, and Anatoly Libgober, Editors,** Singularities and Complex Geometry, 1997

4 **Vyjayanthi Chari and Ivan B. Penkov, Editors,** Modular Interfaces: Modular Lie Algebras, Quantum Groups, and Lie Superalgebras, 1997

3 **Xia-Xi Ding and Tai-Ping Liu, Editors,** Nonlinear Evolutionary Partial Differential Equations, 1997

2.2 **William H. Kazez, Editor,** Geometric Topology, 1997

2.1 **William H. Kazez, Editor,** Geometric Topology, 1997

1 **B. Greene and S.-T. Yau, Editors,** Mirror Symmetry II, 1997

This page intentionally left blank

Some Current Topics on Nonlinear Conservation Laws

This page intentionally left blank

https://doi.org/10.1090/amsip/015

AMS/IP
Studies in
Advanced
Mathematics

Volume 15

Some Current Topics on Nonlinear Conservation Laws

Lectures at the Morningside Center of Mathematics, 1

Ling Hsiao and
Zhouping Xin, Editors

American Mathematical Society • International Press

Shing-Tung Yau, Managing Editor

2000 *Mathematics Subject Classification.* Primary 35–02, 35L65, 35L67;
Secondary 35L60, 35L80, 76N10, 76P05, 46N20, 35Q30.

Library of Congress Cataloging-in-Publication Data

Some current topics on nonlinear conservation laws : lectures at the Morningside Center of Mathematics, 1 / Ling Hsiao and Zhouping Xin, editors.
p. cm. — (AMS/IP studies in advanced mathematics, ISSN 1089-3288 ; v. 15)
Includes bibliographical references
ISBN 0-8218-1965-8
1. Conservation laws (Mathematics) 2. Nonlinear theories. I. Hsiao, Ling, 1940– II. Xin, Zhouping, 1959– III. Series.
QA377.S665 2000
515′.353—dc21 00-025164

Visit the AMS home page at URL: http://www.ams.org/
Visit the International Press home page at URL: http://www.intlpress.com/

10 9 8 7 6 5 4 3 2 1 05 04 03 02 01 00

Contents

Series Preface

This is a subseries of AMS-IP publications dedicated to the research activities of the Morningside Center of Mathematics, Chinese Academy of sciences in Beijing, China. The Morningside Center supports research activities by inviting leading figures in different disciplines of mathematics to give lectures. These leaders direct programs in their disciplines. The participants are young Chinese mathematicians or some visitors from overseas. At the end of the programs, the lectures are collected and organized by the program directors. We believe these programs have been very successful and helpful in promoting mathematics as a whole.

We wish to thank the Morningside Foundation and the Chinese Academy of Sciences for providing the funding. We also thank the staff at our Morningside Center to help in our programs. We also like to take this opportunity to thank all the universities in China who have cooperated with us on these activities. Most of all, to the mathematicians all over the world who have provided unselfish help. We thank them for their efforts. We would also like to thank the International Press and American Mathematical Society for the help of publication.

S.T.Yau and YANG Lo

This page intentionally left blank

PREFACE

A year-long program on nonlinear conservation laws was organized by us at the Morningside Center of the Academic Sinica in Beijing during the academic year of 1997. The aim was to introduce young researchers in China to the frontiers of an important branch of applied mathematics that has been expanding rapidly. The program consisted of mini-courses on various current topics, short lecture series, invited talks, member seminars, and intensive working seminars. The emphasis was laid on theoretical analysis and applications. Important aspects, numerical methods for large scale discontinuous flows, were left for a future program. The program's success is partially reflected from the quality of this volume which consists of selected lecture notes for the mini-courses. The topics covered in this volume include: theory of L^1-well-posedness for system of conservation laws by A. Bressan, theory of compartness methods by G. Chen, mathematical modelling of semi-conductor devices by P. Degond, kinetic formulation of conservation laws by B. Perthame, theory of viscous conservation laws by Zhouping Xin, and mathematical theory of ideal incompressible fluid by Yuxi Zheng. The volume also includes a survey article by Zhouping Xin.

The success of such an undertaking is due to the generous support of the Morningside Center of the Academic Sinica and the help of many individuals. We are particularly grateful to Professors S. T. Yau and L. Yang for their interests in this program and their support; to Professor Z. J. Lu for his constant practical support and help; and to Professor A. Bressan, G. Chen, P. Degond, B Perthame, and Y. Zheng for their lectures and contributions to this volume. We also wish to thank the junior participants in the program, Cheng He, Hailiang Li, R. H. Pan, Shaoqiang Tang, K. Zhang, Ping Zhang, P. Zhu, etc, especially Hai Liang Li, who have made various contributions to the success of the program and this volume.

Ling Hsiao and Zhouping Xin

AMS/IP Studies in Advanced Mathematics
Volume **15**, 2000

Some Current Topics in Nonlinear Conservation Laws

Zhouping Xin
Courant Institute of Mathematical Sciences
New York University
251 Mercer Street
New York City, NY 10012 USA
and
IMS, and Department of Mathematics
The Chinese University of Hong Kong
Shatin, N.T., Hong Kong

Many basic phenomena in continuium mechanics and various other branches of natural sciences are governed by nonlinear conservation laws of the form [33, 87, 178]:

$$\partial_x u + \nabla_x \cdot F(u) = \varepsilon A(u), \qquad u \in \mathbb{R}^n, x \in \mathbb{R}^m, \tag{0.1}$$

and its variants by taking into account other additional physical effects such as dispersion, relaxations, chemical reaction, and external sources, etc, besides the convection, compressions, and dissipation, where $\varepsilon \geq 0$, A is some elliptic operator, and $F(u) = \big(f_1(u), \cdots, f_m(u)\big)$ is a smooth nonlinear map. The convection is assumed to be hyperbolic in the sense that the $n \times n$ matrix $\xi \cdot \nabla_u F(u)$ has n real eigenvalues for all $\xi \in \mathbb{R}^m \setminus \{0\}$. The most important examples of (0.1) may be the following Navier-Stokes equations and many of their variants [33,87,174]:

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) = 0 \\ \partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) + \nabla p = \operatorname{div}(T) \\ \partial_t(\rho E) + \operatorname{div}(\rho u E + up) = \operatorname{div}(uT) + k\Delta\theta \end{cases} \tag{0.2}$$

which express the fundamental physical laws in continuium mechanics: conservation of mass, momentum, and energy, where $x \in \mathbb{R}^d, t \in \mathbb{R}^1$; the unknown, ρ, $u \in \mathbb{R}^d, p, e$ and θ denote the density, pressure, internal energy, and temperature respectively while E and T represent the total energy and stress tensor given by $E = \frac{1}{2}|u|^2 + e$

and $T = \mu\big(\nabla u + (\nabla u)^t\big) + \mu'(\mathrm{div} u)I$ with μ and μ' being the first and second coefficients of viscosity respectively, and I is the $n \times n$ identity matrix; k is the coefficient of heat conduction. It is assumed that $\mu \geq 0$, $\mu' + \frac{2}{d}\mu \geq 0$, and $k \geq 0$. The relation between ρ, p, θ, and e is given by the equation of state for the fluid concerned and the second law of thermodynamics. For ideal inviscid fluids, $\mu' = \mu = k = 0$, then (0.2) becomes the well-known compressible Euler equations:

$$\begin{cases} \partial_t \rho + \mathrm{div}(\rho u) = 0 \\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u) + \nabla p = 0 \\ \partial_t(\rho E) + \mathrm{div}(\rho u E + up) = 0 \end{cases} \tag{0.3}$$

which is one of the most important examples of systems of nonlinear hyperbolic conservation laws, which have been the main focus and the driving forces for mathematical theory of shock waves in the last centrary [33, 90, 178, 159]. Other examples of (0.1) include: the Euler and Navier-Stokes equations for incompressible fluids, the equations of elasticity, the equations of electro-magneto field dynamics for electrically conducting compression fluids, and equations for combustion, oil reservoir simulations, multiphase flows, etc. [87, 33, 178, 131, 129, 53, 135].

The most important feature of system (0.1) is that the speed of wave depends on the wave itself, which leads to great complexity and rich phenomena in the behavior of solutions to (0.1) [33, 178]. This can be revealed by understanding the dynamics and interactions of basic linear and nonlinear waves such as shock waves, rarefaction waves, solitons, diffusion waves, vortex sheets, and boundary layers, etc. [90, 129] The study on the properties of the solutions to (0.1) has important applications to many fields such as turbulence theory, geophysics, meteorology, material sciences, multi-phase flows, and chemical engineering, etc. This poses many problems which are both physically important and mathematically challenging.

Substantial progress has been made in solving these nonlinear conservation laws both theoretically and numerically, and in understanding and interpreting the behaviors of solutions to these systems by the important analytical works [90, 33, 50, 35, 40, 41, 55, 56, 97, 98, 102-104, 128, 141, 157, 159, 165-167, 13, 109, 57, 60, 162, 66, 2-3, 73, 16, 38, 86, 129, 136, 195, 32, 17], and many powerful modern high resolutions-numerical methods for calculating discontinuous flows (or waves with large gradients) have been developed [31, 63, 39, 147, 148, 70-72, 6, 7, 91, 93, 171]. This is particularly so in the one-space-dimension problems, and for suitably "small" solutions with some notable exceptions [4, 39, 98, 100, 85, 165]. Fundamental concepts such as basic linear and nonlinear waves, physical and Lax's geometrical entropy conditions, Riemann solutions, loss of regularity, dissipature mechanism of nonlinearity, compactness and irreversibility of the solution operator, etc, had been established for strictly hyperbolic systems (in one space dimension) almost three decades ago by the pioneering works in [50, 88-90, 159, 178]. A rather complete and satisfactory theory exists for scalar conservation laws in arbitrary space dimensions in terms of well-posedness theory, regularity and compartness of solution operator, large time asymptotic behavior toward nonlinear waves and local structure of entropy weak solutions, and convergence of various approximate solutions generated by either physical perturbations (such as viscous, relaxational, and kinetic extensions, see the lectures by Chen, Perthame, and Xin in this volume and the references therein), or various numerical schemes [85, 93, 122,127, 142, 165, 168, 175, 76]. All the methods working in this case rely essentially on the availability of the maximum principle in this case. Thus the corresponding theory for systems of nonlinear conservation laws is extremely difficult. For some special systems where a weak maximum principle exists, the existence of weak solution in the space of bounded measurable functions had been proved by the theory of compensated-compactness (see the lectures by Chen and Perthame in this volume for details, also [77, 78, 133, 134, 41, 26, 27]). However, though this elegant approach requires no restrictions on the sizes of the initial (or boundary) data, it does demand that there exist sufficiently many entropy-entropy flux pairs, whose existence is only quaranteed for 2 x 2 systems [88, 155]. Furthermore, relatively little information can be obtained for such weak solutions obtained by the weak convergence methods compared with the ones obtained by the Glimm's method described below.

The most important breakthrough in the mathematical theory of shock waves is the celebrated random choice method pioneered by J. Glimm [55], which yields not only the existence of BV (bounded total variation) weak solutions for general l-D strictly hyperbolic systems of conservation laws with small BV data, but also precise asymptotic structures (both local and in large time) of such weak solutions [56, 36, 40, 104]. The Glimm theory is based on the fact that in one-space dimension, dialation invariant solutions, (Riemann solutions), which are superpositions of elemantary waves: shock waves, rarefaction waves, and contact discontinuities, dominate the asymptotic behavior of general flows, both locally and for large time, and thus form building blocks for general weak solutions. The deep understanding on the propogation and interactions of elemantary waves provides the insight of the construction of the now well-known Glimm's functional which is decreasing in time on the Glimm's approximate solutions consisting of piecewise Riemann solutions, as a consequence of the fact that the possible increase of the total wave strength is always balanced out by the decrease of the total future wave interaction potential. [55, 159]. Similar ideas play crucial roles in the later major refinements and improvments such as, the deterministic version of the Glimm scheme [103, 104], L^∞-norm estimate of the Glimm method [195, 167], and L^1-stability theory of the Glimm's solutions [12, 13, 109]. Tremendous progress has been achieved on the structure, regularity, and large time asymptotic behavior of the Glimms solutions [12, 42, 56, 36, 104]. In particular, recently, Bressan successfully proved the uniqueness and L^1-continuous dependence in the class of the viscosity solutions by a semigroup approach in a weighted space, this important recent development is discussed in detail in the lectures given by Bressan in this volume. More recently, the L^1-continuous dependence of Glimm's solutions is shown by Liu and Yang by constructing a different interesting functional [109].

However, it seems that Glimm's method is limited to strictly hyperbolic systems in one-space-dimension since BV space is not a suitable solution space for hyperbolic system in more than one space dimensions [151]. Multi-dimensional theory for hyperbolic systems of conservation laws still has a long way to be developed with some notable exceptions: the short time structural stability of multi-dimensional gas dynamical shock fronts and rarefaction waves has been established [128, 131]; the theory of weakly nonlinear geometric optics has been systematically developed and applied to many interesting physical problems, such as reflection of shock wave by a wedge, and chemical reaction fluids, etc. (see [131, 68] and references therein); even rigorous justification of the asymptotic theory for resonant wave interactions has been achieved for some interesting cases [74, 131], the development of singularities in finite time from smooth data for Euler system (0,3) has been proved and the structure as shock wave has been verified in many important cases (such as irrotational flows) by the recent theory of geometrical blow-up for hyperbolic system in [2, 3, 156, 194]; and there have been many interesting studies on special multi-dimensional problems with various symmetries [26, 28, 59, 193].

In the last two decades or so, there have been interesting works by many people on the studies of non-strictly hyperbolic systems and equations that change types, which arise as models in Van der Waal gases, elastic-plastic materials, magneto-hydro dynamics, multi-phase flows, and phase transitions [69, 82, 48, 107, 153, 157]. New types of shock waves are needed to solve Riemann problems [69, 153, 107]. These new shock waves behave drastically different from the classical shock waves, and their admissibility and nonlinear stability have been investivgated recently for some prototype models [48, 49, 106, 112], where new wave phenomena have been found. Even in the cases that the Riemann problems can be solved successfully, it seems that the great complexity in the interactions of elementary waves makes it almost impossible to apply Glimm's approach in general except in the case of small perturbation of strong waves [153]. For certain 2 x 2 systems, the theory of compensated compactness has been applied to obtain the general weak solutions [27, 78, 141]. A new approach seems to be desired to treat general non-strictly hyperbolic systems.

The inviscid Euler system (0.3) is expected to describe the large scale structures of general viscous flows which are governed by the more physical Navier-Stokes Equations. The studies on the general Navier-Stokes equations have been the center of the continuum mechanics. Two major issues on compressible Navier-Stokes equations have been investigated extensively in the past decades. One is the global (in time) well-posedness of smooth solutions and the large time asymptotic behavior for multi-dimensional compressible Navier-Stokes system with fixed viscosity and heat conduction which is of hyperbolic-parabolic type. This has been succesfully proved in the case for small smooth non-vacuum data whose long time behavior is shown to be governed by the linear diffusion waves [136, 159]. This theory has been generalized even for small discontinuous initial data in multi-dimensions in [64]. The other issue is to understand the effects of small scale dissipations on the behavior of large scale physical flows, i.e. to discribe the asymptotic relationship between the inviscid and viscous flows, in the presence of discontinuities and physical boundaries for either small (but non-zero) dissipation or large time (with fixed dissipation). This is a very important issue not only for the understanding of the asymptotic behavior of the solutions to the Navier-Stokes system, but also crucial for the theory of inviscid hyperbolic systems, which is due to the fact that it is the dissipation mechanism which selects the correct weak solutions to the inviscid system from other nonphysical solutions. Other considerations such as entropy production, entropy inequalities, or even the existence of visions shocks profile do not suffice, as examples in [48, 66] show. This is especially important for non-strictly hyperbolic problems, problems in multi-dimensions and with boundaries. There has been a great success achieved in this direction in terms of nonlinear and linear waves in the past decade. Indeed, an almost complete theory on the nonlinear large time asymptotic stability toward basic viscous waves (viscous shock profiles, rarefaction waves, viscous contact waves, and diffusion waves) has been developed for quite general systems of viscous, strictly hyperbolic conservation laws [57, 80, 105, 108, 110-111, 137, 162, 163, 183, 186-187, 198-199], and even for some pro-type non-strictly hyperbolic models with viscosity [29, 49, 106, 112]. Some of these results have been generalized to multi-dimension for scalear equations and some special systems [58, 185]. Furthermore, many new interesting methods, such as rigorous mulit-scale matched asymptotic method, weighted-characteristic energy method, and approximate paramatric method, etc., which build more hyperbolic elements into the conventional methods by taking into account more internal structure of the underlying flows, have been developed, and should be very useful for other problems. The insights gained in the nonlinear stability theory can be crucial in the studies of a related long standing problem - zero dissipation problem, which is to show that the regular solutions to the visious system converge (in a suitable topology) to entropy weak solutions of the corresponding inviscid system as the dissipation goes to zero, and to understand the structures and dynamics of the viscous solutions across discontinuities and boundaries for small but non zero viscosity. The study of this problem has been the main driving force for both theoretical analysis and numerical methods for scalar conservation laws [85, 93, 159]. However, major difficulties occur for general systems due to the lack of compactness of the viscous solutions except for some special 2 x 2 systems where some weak maximum principle is available so that the theory of the compensated compactness applies to show the convergence of viscous solutions to weak solutions to the coresponding inviscial systems, see the lectures by Chen and Perthame in this volume for details, also in [141]. In the past several years, a new approach, which is based on a multi-scale matched asymptotic analysis and a-priori estimates motivated by the theory of nonlinear stability of viscous waves, has been developed to treat specific but physically interesting flows for general systems [60, 65, 181, 188, 190]. In particular, it is shown in [60] that piecewise smooth solutions with entropy-satisfying shock discontinuities to general strictly hyperbolic systems can be realized as limits of viscous solutions as viscosities tend to zero, and detailed local structures of the viscous solutions have been obtained, see also [196] for recent improvement. For more ideas on this approach, we refer the reader to the lectures by Xin in this volume. This approach has been applied to treat other singular perturbation problems [168]. The zero dissipation limit problem becomes more singular in the presence of physical boundaries due to the appearance of boundary layers. There have been extensive studies on the boundary layer theory due to its importance in the applications to high speed flows [154]. Most engineering literature deals with the formal asymptotic analysis of boundary layers for steady flows with some specific flow configurations [154, 47, 147]. In the case where the boundaries are uniformly non-characteristic, the multi-scale structure of the viscous solutions can be revealed by matched asymptotic analysis and the strong convergence of the viscous solutions to the inviscid flow away from the boundaries can be obtained by nonlinear stability analysis of the boundary layer before shock formation provided that the boundary layer is suitably weak [52, 62, 75, 5, 139, 181]. For the well-known non-slip boundary conditions for the Navier-Stokes equations, which is uniformly characteristic, the formal prandtl's boundary layer theory has existed since the early of this century, yet its validity has been proved rigorously for linearied flows in general Sobolev space in [182] and for nonlinear flows only in the analytical setting [152]. The study on the Prandtls boundary layer equations, which is a degenerate parabolic-elliptic system, is quite difficult in general Sobolev spaces. Only some partial results, such as finite time blow-up, long time existence of solutions for special class of monotonic initial data, and steady flows, exist [46, 47, 146].

Another major recent development in the theory of the compressible Navier-Stokes system is the global (in time) existence of large amplitude weak solutions to either the Cauchy problems or initial boundary value problems of the isentropic compressible Navier-Stokes equations with some special equations of state [98]. Although the uniqueness and regularities of such weak solutions remain open, some important physical properties, such as boundedness of the energy and the zero-mach number limit to the solutions of the incompressible Navier-Stokes equations, have been established for these weak solutions [98, 99]. The approach of this theory is a weak convergence method based on the standard energy estimate and an additional space-time higher estimate which is guaranteed by the special form of the equation of state [98]. It is remarkable that such a approach can work thus far since almost all the a -prior estimates required for this theory are well known. It should be noted that in the case where the initial data is not periodic, the existence of weak solutions [98] requires that the far fields must be vacuum, and thus the weak solutions cannot be regular in general even for smooth initial data due to the fact that any non-trival smooth solution with compartly supported initial density must blow-up in finite time [180]. This somewhat surprising fact is a direct consequence of the observation that the pressure behaves dispersively for both Euler and Navier-Stokes systems in arbitrary space dimensions if the far fields are at vacuum [180, 132]. This instance shows the importance of understanding vacuum state in the theory of the compressible fluids besides the physical relevance, such as evolution of gaseous stars due to the high order degeneracy of both Euler and Navier-Stokes systems at vacuum. Some rich phenomena have been revealed in recent studies involving vacuum state, which include the dispersive behavior of the total pressure [180], the singular motions of the free surfaces separating the gases from vacuum, and the better regularity of the inviscid flows than the viscous ones, etc [113, 127].

The compressible Euler or Navier-Stokes systems either give macroscopic descriptions of interacting particles in a fluid motions as the mean free path tends to zero in the kinetic theory [21] or govern the equilibrium states of thermo-non-equilibrium process as the rate of relaxation goes to zero [22]. There have been increasing interests in studing the hydrodynamic limits for various kinetic models and the zero-relaxation limits for many thermo-non-equilibrium processes in the recent years [4, 18-21, 25, 61, 70-72, 81, 94, 101-102, 116-117, 123-124, 127, 133-134, 142-143, 145, 158-149, 157, 174, 176-177, 184, 188-189, 192, 199]. Besides the obvious physical significance of such limits problems, one would also hope that the complexities and many great difficulties such as loss of regularity, non-uniqueness of weak solutions, global entropy condition, and considerable difficulties in numerical calculations, encounted at the macroscopic level, can be illuminated and overcome by looking into the microscopic discriptions of the fluid motions which are not only physically sound but also mathematically tractable. Despite the substantial progress such as Chapman-Enskog theory [61, 21], initial layer theory [20, 18], and existence of shock profiles [19], etc have been achieved for the hydro-dynamic limit problems for various models of the nonlinear Boltzmann type equations, the global nonlinarity of the interaction operators for Boltzmann type transport equations and the stiffness of the hydro-dynamic limit make it extremely difficult (if not impossible) to study the limit rigorously and to gain deep insight about the macroscopic flows, in particular, in the presence of discontinuities and physical boundaries [19, 21, 116-117, 174, 176, 184, 192] One of recent efforts has been concentrated on designing better (in the sense that they are physically reasonable and amiable to mathematical analysis both analytically and numerically) kinetic models or relaxation approximations for a given system of a hyperbolic conservation laws [11, 70-72, 79, 101, 143, 148-149, 192]. One such approach is the so called kinetic formulation of conservation laws, which artificially construct a collision operator in a similar spirit as that for the BGK model of Boltzmann equation [21] so that the Maxwellian satisfies the given conservation laws [149, 101]. This approach was successfully applied to multi-dimension scalar conservation laws and some special systems, for more detailed discussions on this approach, we refer the reader to the lectures by B. Perthame in this volume and the references therein. Another approach is the local relaxations approximation methods introduced recently by Jin and Xin [70]. The basic idea is that for any given general system of nonlinear conservation laws, one can construct a corresponding linear hyperbolic system with a nonlinear stiff source term that approximates the original system of conservation laws with a small dissipative correction under the so-called "subcharacteristic condition" which ensures the relaxation system contains all the information of the original system. The special structure of the relaxation system proposed in [70], such as linear characteristic fields with constant waves speeds; localized nonlinear interaction with relaxational structure; and the stiff source terms being not-fully ranked and linear in the induced variables which approximate the fluxes of the original conservation laws, makes it more advantagerous to solve the relaxation system numerically than the original system of conservation laws. Indeed, the resulting relaxation schemes are high order accurate, entropy preserving, and requiring neither the Riemann solvers nor nonlinear algebraic solvers [70]. The main features of the relaxation schemes in [70-72] are their simplicity and generality. General systems of higher space dimensions can be treated in the same way as in the one dimension. Furthermore, there is no requirement of hyperbolicity in the formulation, thus the relaxation schemes should be useful in many problems such as in MHD, multiple phase flows, and elastic-plastic materials, etc., in which Riemann solvers (or ever approximate Riemam solvers) are not easily available. Various extensions of the local relaxation idea exist [71, 72, 79, 143], and rigorous analysis of the asymptotic equivalence between and relaxation system and the limiting system of conservation laws has been obtained in many interesting cases [123-124, 127, 142, 168, 176, 192].

On the other hand, in many pratical applications, the macroscopic models, are considerably simpler both conceptually and numerically than their macroscopic kinetic models [21, 22, 135, 174], and multi-scale expansion methods, such as Hilbert and Chapman-Enskog expansions are very powerful to derive useful macroscopic models from the more fundamental kinetic descriptions. One particular case is the mathematical modelling of electron transport in semi-conductor materials [135]. A semiconductor is a solid-state material with an intrinsically low electron conductivity. On the quantum level, the state of a free electron is governed by the Schrodinger equation. In the semi-classical limit, the kinetic description of the electron and hole population in a semiconductor device is given by the nonlinear Boltzmann equations governing the distribution functions with the electrostatic potential satisfying a Poission equation. However, such a detailed description for carrier transport in semiconductors is too complicated for pratical purposes, and many numerical codes based on Monte-Carlo or particle methods, are very expensive and cannot be routinely used to design components. Various macroscopic models have been sought. The mostly widely used model in practice is the so called drift-diffusion model, which is a parabolic equation for electron density and has been extensively studied both mathematically and numerically since early fifties [51, 125, 133, 135]. The improvement of the drift-diffusion model is the Energy-Transport model, which consists of a parabolic system for the electron density and energy, and has been derived from the Boltzmann equation by multiscale analysis only recently [8]. The Energy-Transport model, which includes the drift-diffusion model as a particular case, resembles in many ways the compressible Navier-Stokes equations and has been studies extensively recently [8, 37]. These topics and other models are discussed in great detail in the lectures by P. Degond in this volume.

In the limit of small Mach number, one can derive from the compressible Euler and Navier-Stokes systems the incompressible Euler and Navier-Stokes equations respectively [83, 99], which govern various flows such as water in the oceans and air in the atmosphere. The mathematical theory for the incompressible fluid is more mature than that for the compressible one (see [32, 86, 97, 129, 179]). In particular, the well-posedness theory for smooth solutions exists in 2-D for both Euler and Navier-Stokes systems thanks to the fact that the vorticity of the flow satisfies a maximum principle [97, 129]. There have been some signficant results concerning physically important inviscid 2-D singular flows: the motions of vortex patch and vortex sheets [129, 160, 140, 84, 150, 164, 17, 119-120]. For the vortex patch problem, here the vorticity is bounded, the well-posedness is proved long age [197], even the preservation of the regularity of the boundary of the vortex patch has been achieved recently [23, 10]. While the progress has been much slow and limited for the vortex sheets problem, where the vorticity is a finite Radom measure concentrated on a curve which is smooth initially, the global (in time) existence of classical weak solutions, and convergence of approximate solutions generated by either viscous regularization or vortex method have been achieved only recently for the case that the initial vorticity has single sign [38, 44, 115, 118, 130] except for special flows [121], though the short time existence in the analytic setting have been extensively studied [17, 129, 140]. Many important questions such as uniqueness of weak solutions, structure of the vortex sheet after roll-up, and behavior of approximate solutions remain open. For more discussion on this and related problems, we refer to the lectures by Y. Zheng in this volume. Much less progress has been made for 3-D incompressible flows, although the Leray-Hopf weak solutinos to the 3-D Navier-Stokes equation have been shown to exist globally in time for quite general data and exterior forces [32, 86], the partial regularity and large time behavior of the Leray-Hopf weak solutions have been obtained [16, 95, 96, 169, 170], and the existence of large amplitude strong solutions under certain symmetries has been established [173, 86]. Yet, most central questions in the incompressible fluid mechanics, such as whether finite time singularities do develop from smooth initial data for both inviscid and viscous flows, the uniqueness and regularity of Leray-Hopf weak solutions with large amplitudes, long time dynamics of viscous flows, and instability of visious boundary layers, etc., remain completely open and challenge the field for many years to come, though some special types of blow-up for

Navier-Stokes, such as self similar blow-up, has been ruled out recently [144, 170, 172]. Even the existence of weak solution to the inviscid Euler equations has not been proved except the cases with special symmetries [129, 173], and the existence of measure-valued solutions [45].

Another most significant achievement in the past several decades is the development of many powerful modern high resolution numerical methods for calculating large scale flows governed by (0.1) [29, 31, 63, 147, 93, 70, 6, 7, 120, 84]. This is an important but difficult task due to the possible appearance of transitional layers, (such as shock layers, boundary layers, and shear layers etc.) in the flows governed by (0.1). Rich phenomena such as smearing, post-shock oscillations, discrete shock profile, and numerical cell-entropy conditions, etc, have been found. For inviscid flows, the finite difference schemes can be classified into either front-tracking method, or shock-capturing method, and hybridizations of these two [30, 63, 91, 171]. The convergence of many pratical scheme has been proved for scalar equations and some special systems, see [93] and the references therein. However, since most of the lectures in this volume deal mainly with theoretical issues, I will omit the detailed discussions on this important topic.

Despite the important progress achieved in the past on the study of (0.1) and related equations, many fundamental questions remain to be answered and continue to challenge the field for many years to come. Since I have discussed some open problems for (0.1) in great detail in [191], I will sketch some general problems here, and refer the readers to [191] for more discussion on the motivation, related work, and significance of these problems.

1. Well-posedness and local structure of BV weak solutions for 1-D strictly hyperbolic systems (in particular, for the full gas dynamic system) with Cauchy data of arbitrary amplitude. Is the solution unique? Can a weak solution to the full gas dynamics equations in BV space blow-up in finite time [138]? Can a BV solution behave locally as an appropriate peturbation of a Riemann solution as Glimm's solutions do [103]?

2. Local structure and uniqueness of limits of zero dissipation. If a weak solution to the 1-D inviscid strictly hyperbolic system is a limit of solutions to the corresponding viscous system as the viscosity tends to zero, then what is the local structure of such a solution? can it be a viscosity solution defined by Bressan in [12, 13]? Can one show that such a solution is unique in the class of weak solutions obtained as limits of zero dissipation?

3. Spatial periodic solutions and homogenization theory for general 1-D strictly hyperbolic systems. For a given strictly hyperbolic systems without a Riemann invariant coordinates (the 3×3 full gas dynamical system in particular), when do shocks form from a smooth spatial periodic flow [90, 92]? Is there a global weak

solution with periodic initial data? How do the oscillations propogate for such system [56]?

4. Theory of well-posedness of weak solutions to general nonstrictly hyperbolic systems [27, 48, 69, 78, 82].

5. Multi-dimension inviscid shock wave theory. Due to the complexity and lack of understanding, one should concentrate on the multi-dimension compressible Euler system. Despite the recent progress in geometrical blow-up theory [1, 2, 3, 156, 194], the singularity structure of solutions to Euler system has not be found unless the flow is assumed to be irrotational [2, 194]. In general, do shocks form before shell singularities? Can one study some of the special physically relevant wave patterns where a lot of experimental data, numerical simulations, and asymptotic results are available, such as: shock reflection phenomena [9, 54, transonic flows and standing shocks in a nozzle [59], self-similar flows, and other flows with various symmetries [28, 33, 193, 200].

6. Global (in time) well-posedness of the Cauchy problem or initial-boundary value problem for the compressible Navier-Stokes equation with large data in multi-dimension. Can smooth solution develop finite time sigularity [180]? Can the vacuum state be formed in finite for non-vacuum date? Can one obtain regularity for the weak solution constructed by P. L. Lions [98, 180]?

7. Nonlinear stability of planary viscous nonlinear waves for viscous systems is multi-dimension (in particular, for 2D or 3D compressible Navier-Stokes equations). When are basic nonlinear wave patterns: such as planar viscous shock profiles and viscous rarefaction waves, nonlinearly stable under generic multi-dimensional perturbations? What are the large time asymptotic behavior toward a contact wave for solutions to the compressible Navier-Stokes equations?

8.Shock and boundary layer theory for the compressible Navier-Stokes equations. Can one prove the asymptotic equivalence of the Euler and Navier-Stokes systems in the limit of small viscosity and heat conduction in the case that the inviscid flow contains finitely many shock discontinuities [60]? Can one justify the Prandtl's boundary layer theory [182, 154]?

9. 2-D vortex sheets motion. What are the structures of the approximate solutions generated by either Navier-Stokes solutions or vortex methods with general vortex sheets initial data [38, 130, 115, 118]? Doses concentration-cancellation always occur [44]? Does there exist dynamical energy-concentration for such approximate solutions with initial vorticity either one sign or both signs [44]? In the case of bounded domain, can vorticity concentrate near the physical boundary [121]?

10. Fluid-dynamic limits. Study the fluid-dynamic limit problems for general models of the nonlinear Boltzmann equations in the presence of discontinuities and physical boundaries [116-117, 158, 177, 184, 188-189]. What will be the fluid-dynmamic limit for the renormalized weak solutions to the Boltzmann equation obtained by Diperna-Lions in [43]?

References

[1] Alinhac, S., *Explosion geometrique pour der systems quasi-linearies*, Amer. J. Math. 117 (1995).

[2] Alinhac, S., *Temps de vie des solutions regulieres de equations d'Euler compressibles axisymetriques en dimension deux*, Invent. math. 111 (1993), 627-677.

[3] Alinhac, S., *Blowup of small data solutions for a class of quasilinear wave equations in two space dimensions II*, Acta Math. 182(1) (1999), 1-23.

[4] Bardos, C. Golse, F. and Levermore, D., *Fluid dynamic limit of kinetic equations II: convergence proofs for the Boltzmann equation*, Comm. Pure Appl. Math. 46 (1993), 667-753.

[5] Bardos, C. and Rauch, J., *Maximal positive BVP as limit of singular perturbation problems*, Trans. AMS 270, (1982), 372–408.

[6] Beale, T., *The approximation of weak solutions to the 2-D Euler equations by vortex elements*, Multidimensional Hyperbolic Problems and Computations, IMA (1991), 23-37.

[7] Beale, T. and Majda, A., *Vortex Methods II: Higher order accuracy in two and three dimensions*, Math. Comp. 39(1982), 29-52.

[8] Ben Abdalah, N. , Degond, P. , and Genieys, S. , *An energy transport modelfor semiconductors*, J. Math. Phys. 37(1996), 3306-3333.

[9] Ben-Dor, G., *Shock Waves Reflection Phenomena*, Springer-Verlag, New York, 1992.

[10] Bertozzi, A. L. and Constantian, P. , *Global regularity for vortex patches*, Comm. Math. Phys. 152 (1993), 19–26.

[11] Brenier, Y., *Resolution d'equation quasilineaires en deimensions N d'espace a l'aide d'equations lineaires en dimensions N+1*, J. Diff. Eqs. 50(3) (1982), 375-390.

[12] Bressan, A., *The unique limit of the Glimm scheme*, Arch. Rat. Mech. Anal. 130 (1995), 205-230.

[13] Bressan, A., Crasta, G., and Piccoli, B., *Well-posedness of the Cauchy problem for nxn systems of conservation laws*, preprint, 1997.

[14] Bressan, A. and Lefloch, P., *Uniqueness of weak solutions to systems of conservation laws*, Arch. Rat. Mech. Anal. 140 (1997), 301-317.

[15] Bressan, A, Liu, T.P. and Yang, T., *L^1-stabilty estimates for nxn systems of conservation laws*, preprint, 1998.

[16] Caffarelli, L., Kohn, R. and Nirenberg, L., *Partial regularity of suitable weak solutions to the Navier-Stokes Equations*, Comm. Pure Appl. Math. 35 (1982), 771-837.

[17] Caflisch, R. and Orellana, O., *Singularity formation and ill-posedness of vortex sheets*, SIAM J. Numer. Anal. 20 (1988) 1060-1080.

[18] Caflisch, R., *The fluid dynamic limit of the nonlinear Boltzmann equation*, Comm. Pure Appl. Math. 5 (1980), 651–666.

[19] Caflisch, R. and Nicolaenko, B.,*Shock profile solutions of the Boltzmann equation*, Comm. Math. Phys. 86 (1982), 161–194.

[20] Caflisch, R.and Papanicolaou, G., *The fluid dynamical limit of the a nonlinear Boltzmann equation*, Comm. Pure Appl. Math. 32 (1979), 589–616.

[21] Cercignani, C., *The Boltzmann Equation and Its Applications*, Springer-Verlag, New York, 1988.

[22] Chapman, J.S. and Cowling, T., *The Mathematical Theory of Non-Uniform Gases*, Cambridge University Press, 1970.

[23] Chemin, J. Y. , *Sur le Movement des partiacles d'un fluid parfait incompressible bidimensionalel*, Invent. Math. 103 (1991), 599-629.

[24] Chen, G., *Theory of Compensated Compactness and Applications*, preprint, 1988.

[25] Chen, G., Levermore, C. d. and Liu, T. *Hyperbolic conservation laws with stiff relaxation terms and entropy*, Comm. Pure Appl. Math. 47 (1994), 787-830.

[26] Chen, G. and Glimm, J., *Global solution to the compressible Euler equations with geometrical structures*, Comm. Math. Phys. 179(1996), 153-193.

[27] Chen, G. and Kan, P. T., *Hyperbolic conservation laws with umbilic degenracy (I)*, Arch. Rat. Mech. Anal. 130 (1995), 231-276.

[28] Chen, S.X., *Supersonic flow past a non-symmetric conical body*, preprint, 1999.

[29] Chern, I-L., *Multi-mode diffusion waves for viscous non-strictly hyperbolic conservation laws*. Comm. Math. Phys. 138 (1991), 51–61.

[30] Chern, I-L., Glimm, J., McBryan, O., Plohr, B. and Yaniv, S., *Front tracking for gas dynamics*, J. Comp. Phys. 62 (1986), 83–110.

[31] Chorin, A. J., *Numerical study of slightly viscous flows*, J. Fluid Mech. 57(1973), 785-796.

[32] Constantin, P. and Foias, C., *Navier-Stokes Equations*, Lecture in Mathematics, Chicago, The University of Chicago Press, 1988.

[33] Courant, R. and Friedrichs, K. O., *Supersonic Flow and Shock Waves*, Wiley-Interscience, 1948.

[34] Dafermos, C. M., *Large time behavior of solutions of hyperbolic balance laws*, Bull. Soc. Math. Greece (N.S.) 25 (1979), 521–554.

[35] Dafermos, C. M., *Generalized characteristics in hyperbolic systems of conservation laws*, Arch. Rat. Mech. 107 (1989), 127–155.

[36] Dafermos, C. M., *Large time behavior of periodic solutions of hyperbolic systems of conservation laws*, J. Diff. Eqs. 121 (1995), 183-202.

[37] Degond, P. , Genieys, S. , and Jungel, A. , *A system of parabolic equations in nonequilibrium thermodynamics including thermal and electrical effects*, J. Math. Pures Appl. 76(1997), 991–1015.

[38] Delort, J. M. , *Existence de nappes de tourbillon en dimension deux*, J. Amer. Math. Soc. 4(1991), 553-586.

[39] Ding, X., Chen, G. and Luo, P., *Convergence of the Lax-Friedrichs schemes for isentropic gas dynamics I, II*, Acta Mathematica Scientia Vol. 7(4) (1987), Vol. 8(1) (1988).

[40] DiPerna, R., *Uniqueness of solutions to hyperbolic conservation laws*, Indiana Univ. Math. J. 28 (1979), 224–257.

[41] DiPerna, R., *Convergence of the viscosity method for isentropic gas dynamics*, Comm. Math. Phys. 91 (1983), 1–30.

[42] Diperna, R. , *Singularities of solutions of nonlinear hyperbolic systems of conservation laws*, Arch. Rat. Mech. Anal. 60 (1975), 75–100.

[43] Diperna, R. and Lions, P. L., *On the Cauchy problem for Boltzmann equations: Global existence and weak stability*, Ann. Math. ,130 (1989), 321-136.

[44] Diperna, R. and Majda, A., *Concentrations in regularizations for 2-D incompressible flow*, Comm. Pure Appl. Math. 40 (1987), 301–345.

[45] Diperna, R. and Majda, A., *Osillations and concentrations in weak solutions of the incompressible fluid equations*, Comm. Math. Phys. 108 (1987), 667–689.

[46] E, W. and Engquist, B., *Blow-up of solutions of the unsteady equations*, Comm. Pure Appl. Math. 50 (1997), 1287-1293.

[47] Fife,P., *Considerations regarding the mathematical basis for the Prandtl's boundary layer theory*, Arch. Rat. Mech. Anal. 38 (1967), 184-216.

[48] Freistuler, H., *Dynamical stability and vanishing viscosity, a case study of a nonstrictly hyperbolic system*, Comm. Pure Appl. Math. 45(1992), 561-582.

[49] Freistuler, H.,and Liu, T., *Nonlinear stability of overcompressive shock waves in a rotationally invariant system of viscous conservation laws*, Comm. Math. Phys. 153 (1993), 147-158.

[50] Friedrichs, K. and Lax, P. D., *Systems of Conservation laws with a convex extension*, Proc. Nat. Acad. Sci. U.S.A., 68 (1971), 1686–1688.

[51] Gamba, I., *Stationary transonic solutions of a one-dimensional hydrodynamic model for semiconductors*, Comm. Partial Diff. Eqs. 17(1992), 553-577.

[52] Gisclon, M. and Serre, D., *Study of boundary conditions for a strictly hyperbolic systems via parabolic approximation*, C. R. Acad. Sci. Paris Ser. I Math. 319 (1994), no. 4, 377-382.

[53] Glassman, I., *Combustion*, second edition, Academic Press, New York (1981).

[54] Glaz, H., Collela, P., Glass, I. I. and Deschembault, R., *A detailed numerical graphical, and experimental study of oblique shock wave reflections*, Lawrence Berkeley Report (1985).

[55] Glimm, J., *Solution in the large for nonlinear hyperbolic systems of equations*, Comm. Pure Appl. Math. 18(1965), 95–105.

[56] Glimm, J. and Lax, P. D., *Decay of solutions of systems of nonlinear hyperbolic conservation laws*, AMS Memoir 101 (1970).

[57] Goodman, J., *Nonlinear asymptotic stability of viscous shock profiles for conservation laws*, Arch. Rat. Mech. Anal. 95 (1986), 325–344.

[58] Goodman, J., *Stability of viscous scalar shock fronts in several dimensions*, Trans. AMS 311 (1989), 683–695.

[59] Goodman, J., Embid, P. and Majda, A., *Multiple steady states for 1D transonic flow*, SIAM J. Sci. Statist. Comput. 5 (1984), 21–41.

[60] Goodman, J. and Xin, Z. P., *Viscous limit to piecewise smooth solutions of systems of conservation laws*, Arch. Rat. Mech. Anal. 121(1992), 235-265.

[61] Grad, H., *Singular and nonuniform limits of the Boltzmann equation*, SiAM–AMS Proc. I, Transport Theory (1968), 296-308.

[62] Grenier, E., and Gues, O., *On the inviscid limit of noncharacteristic nonlinear parabolic systems*, preprint, 1998.

[63] Harten, A. , Engquist, B. , Osher, S. , and Chakravarthy, S. , *Uniformly high order accurate essentially non-oscillatory schemes III*, J. Comput. Phys. 71 (1987), 231–303.

[64] Hoff. D., *Global solution of the Navier-Stokes Equations for multidimensional compressible flow with discontinuous initial data*, J. Diff. Equns. 120 (1995), 215-254.

[65] Hoff, D., and Liu, T., *The inviscid limit for the Navier-Stokes equations of compressible, isentropic flow with shock data*, Indiana Univ. J. 38(4) (1989), 861-915.

[66] Hsiao, L., *The entropy rate admissibility criterion for gas dynamics*, J. Diff. Eqs. 38 (1980), 226-238.

[67] Hsiao, L. and Luo, T., *Nonlinear diffusive phenomena of solutions for the system of compressible adiabatic flow through porous media*, J. Diff. Eqs. 125 (1996), 329-365.

[68] Hunter, J. *Strongly nonlinear hyperbolic waves*, Nonlinear Hyperbolic Equations-Theory, Computational Methods, and Applications, J. Ballman, etc. (eds.), Viewig, 1989, 257-268.

[69] Isaacson, E. , Marchesin, D. , and Plohr, B. , *Transitional waves for conservation laws*, SIAM J. Math. Anal. 21(1990), 837–866.

[70] Jin, S. and Xin, Z. P., *The relaxation schemes for systems of conservation laws in arbitrary space dimensions*, Comm. Pure Appl. Math. 48(1995), 235-277.

[71] Jin, S., and Xin, Z. P., *Numerical passage from systems of conservation laws to Hamilton-Jacobi equations*, preprint, to appear at SIAM J. Num. Anal. 1998.

[72] Jin, S., Katsoulakis, M., and Xin, Z. P., *Relaxation schemes for curvature-dependent front propogation*, preprint, to appear at Comm. Pure Appl. Math. 1998.

[73] John, F., *Formation of singularities in one-dimensional nonlinear wave propagation*, Comm. Pure Appl. Math. 27 (1974), 337-405.

[74] Joly, J. L. , Metivier, G. , and Rauch, J. , *Focusing at a point and absorption of nonlinear oscillations*, Trans. AMS 347(1955), 3921–3970.

[75] Joseph, K. T., *Boundary layers in approximate solutions*, Transac. AMS 314 (1989), 709–726.

[76] Joseph, K. T., *Burgers's equation in the quarter plane, a formula for weak limit*, Comm. Pure Appl. Math. XLI (1988), 133–149.

[77] Kan, P. , M. Santos, M. , and Xin, Z. P. *Initial- boundary value problems for conservation laws*, Comm. Math. Physics, **186** (1997), 701-730.

[78] Kan, P. , Santos, M. , and Xin, Z. P., *Initial boundary value problem for a class of quadratic systems of conservation laws*, Mathematica Contemporanea, 11 (1996), 1–32.

[79] Katsoulakis, M. A. and Tzavaras, A. E. , *Contractive relaxation systems and the scalar multidimensional conservation laws*, Comm. PDEs. 22 (1997), 195–223.

[80] Kawashima, S. and Matsumura A.,*Asymptotic stability of travelling wave solutions of systems of one-dimensional gas motion.* , Comm. Math. Phys. 10 (1985), 97–127.

[81] Kawashima, S., Matsumura, A. and Nishida, T., *On the fluid dynamical approximation to the Boltzmann equation at the level of the Navier-Stokes equations*, Comm. Math. Phys. 70 (1979), 97–124.

[82] Keyfitz, B. and Kranzer, H., *A system of nonstrictly hyperbolic conservation laws arising in elasticity theory*, Arch. Rat. Mech. Anal. 72 (1980), 219–241.

[83] Klainerman, S and Majda, A., *Compressible and incompressible fluids*, Comm. Pure Appl. Math. 35(1982), 632–653.

[84] Krasny, R. , *Computation of vortex sheets roll-Up in the Trefftz plane*, J. Fluid Mech. , 184 (1987), 123-155.

[85] Krushkov, S.N., *First-order quasilinear equations with several spacve variables*, Mat. Sb. 123 (1970), 228–255.

[86] Ladyzenskaya, O. A., *The Mathematical Theory of Viscous Incompressible Flow*, New York , Gordon and Breach Science Publishers, 1969.

[87] Landau, L. and Lifschitz, E., *Fluid Mechanics*, Addison-Wesley, New Yor5k, 1953.

[88] Lax, P. D., *Shock waves and entropy. In Contributions to Nonlinear Functional Analysis*, Editted by E. Zarantinello. Academic Press: New York, 1971, 603–634.

[89] Lax, P. D., *Hyperbolic systems of conservation laws II*, Comm. Pure Appl. Math. 10 (1957), 537–566.

[90] Lax, P. D., *Hyperbolic Systems of Conservation Laws And The Mathematical Theory of Shock Waves*, Conf. Board Math. Sci. 11, SIAM, 1973.

[91] Leer, B. van, *Towards the ultimate conservative difference schemes V. A second order sequel to Godunov's method*, J. Comp. Phys. 32 (1979), 101–136.

[92] Lefloch, P. and Xin, Z. P., *Formation of singularity in periodic solutions to gas dynamic equations*, preprint.

[93] LeVeque, R. J. , *Numerical Methods for Conservation Laws*, Birkhauser Verlag, Basel (1990).

[94] Levermore, C. D., *Moment closure hierarchies for kinetic theories*, J. Stat. Phys. 83(1996), 1021–1065.

[95] Lin, F. H., *On the partial regularity of Navier-Stokes equations*, preprint, 1998.

[96] Lin, F. H. and Lu, C., *Partial regularity of the dynamical system modelling the flow of liquid crystals*, Discrete Contin. Dynam. System, 2(1996), 1–22.

[97] Lions, P. L., *Mathematical Topics in Fluid Mechanics*, Vol. 1, Incompressible Models. Lecture Series in Mathematics and Its Applications, V. 3, Clarendon Press, Oxford, 1996.

[98] Lions, P. L., *Mathematical Topics in Fluid Mechanics*, Vol. 2, Compressible Models, Lecture Series in Mathematics And Its Applications, V. 6, Clarendon Press, Oxford, 1998.

[99] Lions, P. L. and Masmudoi, N., Incompressible limit for a viscous compressible fluid, J. Math. Pures Appl. 77 (1998), 585-627.

[100] Lions, P. L., Perthame, B., and Souganidis, P.E., *Existence and stability of entropy solutions for the hyperbolic systems of isentropic gas dynamics in Eulerian and Lagrangian coordinates*, Comm. Pure Appl. Math., 49 (1996), 599-638.

[101] Lions, P. L., Perthame, B. and Tadmor, E., *Kinetic formulation of the isentropic gas dynamics and p-system*, Comm. Math. Phys. 163 (1994), 415-431.

[102] Liu, T.P. *Hyperbolic conservation laws with relaxation*, Comm. Math. Phys. 108(1987), 153–175.

[103] Liu, T. P., *The deterministic version of the Glimm scheme*, Comm. Math. Phys. 57(1977), 135–148.

[104] Liu, T. P., *Admissible solutions of hyperbolic conservation laws*, AMS Memoirs 240 (1981).

[105] Liu, T. P., *Pointwise convergence to shock waves for systems of viscous conservation laws*, preprint, to appear at Comm. Pure Appl. Math. (1985).

[106] Liu, T. P. and Xin, Z. P., *Stability of viscous shock waves associated with a system of nonstrictly hyperbolic conservation laws*, Comm. Pure Appl. Math. 45(1992), 361-388.s

[107] Liu, T. P. and Xin, Z. P., *Overcompressibve shock waves*, Nonlinear Evolution Equations That change Type, IMA Volumes in Mathematics And Its Applications, V. 27, Edited by Keyfitz and Shearer. Springer-Verlag, New York-Heidelberg-Berlin, (1991), 139–145.

[108] Liu, T. P. and Xin, Z. P., *Pointwise decay to contact discontinuities for systems of conservation laws*, Asia J. Math., 1(1997), 701-730.

[109] Liu, T. P. and Yang, T. *L_1-well-posedness of Glimm solution for systems of conservation laws*, preprint, 1998.

[110] Liu, T. P. and Xin, Z. P., *Nonlinear stability of rarefaction waves for compressible Navier-Stokes equations*, Comm. Math. Phys. 118 (1988), 451–465.

[111] Liu , T. P. and Zeng, Y., *Large time behavior of general systems of hyperbolic-parabolic conservation laws*, AMS Memoir, 1997.

[112] Liu, T. and Zumbrun, K., *Nonlinear stability of an undercompressive shock for complex Burger equation*, Comm. Math. Phys. 168 (1995), 163-186.

[113] Liu, T. P. , Xin, Z. P., and Yang, T., *On vacuum states for isentropic compressible flow*, Disc. Con. Dynamical Systems, 4 (1998), 1-32.

[114] Liu, J. and Xin, Z. P., *Nonlinear stability of discrete shocks for systems of conservation laws*, Arch. Rat. Mech. Anal. 125 (1993), 217-256.

[115] Liu, J. and Xin, Z. P., *Convergence of Vortex Methods for weak solutions to the 2-Euler Equations with vortex sheet data*, Comm. Pure Appl. Math. 48 (1995), 235-277.

[116] Liu, J. and Xin, Z. P., *Boundary layer behavior in the fluid-dynamic limit a model Boltzmann equation*, Arch. Rat. Mech. Anal. 135 (1996), 61-105.

[117] Liu, J. and Xin, Z., *Kinetic and viscous boundary layers for Broadwell equations*, Transport Theory and Statistical Phys. 25 (1996), 447-462.

[118] Liu, J. and Xin, Z. P., *Convergence of point vortex method for 2-D vortex sheets*, preprint, to appear at SIAM J. Numer. Anal. (1998).

[119] Liu, J. and Xin, Z. P., *Convergence of a Galerkin method for 2-D discontinuous Euler flows*, preprint, to appear at SIAM J. Numer. Anal. (1998).

[120] Liu, J. G. and Shu, C. W., *A discontinuous Galerkin method for 2-D Euler flows*, preprint, 1998.

[121] Lopes, H. J. Lopes, M. C. and Xin, Z. P., *Existence of vortex sheets with reflection symmetry in two space dimensions*, Preprint, 1999, to appear in Amer. J. Math. .

[122] Lucier, B., *Regularity through approximation for scalar conservation laws*, SIAM J. Math. Anal. 19 (1988), 763–777.

[123] Luo, T., *Asymptotic stability of planar rarefaction waves for the relaxation approximation of conservation laws in several dimensions*, J. Diff. Eqs. 133 (1997), 255–279.

[124] Luo, T. and Xin, Z. P. *Nonlinear stability of shock fronts for a relaxation system in several space dimensions*, J. Diff. Equations, Vol. 139(1997), 365-408.

[125] Luo, T., Natalini, R. and Xin, Z. P. *Large time behavior of solutions to a hydrodynamic model for semi-conductors*, SIAM J. Applied Math. 59 (1999), No. 3, 810–830.

[126] Luo, T., Xin, Z. P. , and Yang, T. *Interface behavior of compressible Navier-Stokes equations*, to appear at SIAM J. Applied Math.(1998).

[127] Luo, T. Xin, Z. P. , and Yang, T., *Boundary entropy conditions for hyperbolic conservation laws with relaxation approximation*, preprint, 1999.

[128] Majda, A., *Existence of multidimensional shock fronts*, Mem. AMS 281 (1983).

[129] Majda, A. and Bertozzi, M., *Vorticity and the Mathematical Theory of Incompressible Fluid Flow*, 1997.

[130] Majda, A., *Remarks on weak soluiotns for vortex sheets with a distinguished sign*, Indiana Univ. Math. J. 42 (1993), 921-939.

[131] Majda, A., *Compressible Fluid Flow and Systems of Conservation Laws*, Applied Mathematical Sciences, 53, Springer-Verlag, New York, 1984.

[132] Makino, T., Ukai, S. and Kawashima, S., *Sur solutions a support de L'equation d' Euler Compressible*, Japan J. Appl. Math. 3 (1986), 249–257.

[133] Marcati, P. and Natalini, R. , *Weak solutions to a hydrodynamic model for semiconductors and relaxation to the drift-diffusion equation*, Arch. Rat. Mech. Anal. 129 (1995), 129–145.

[134] Marcati, P. and Rubino, B., *Hyperbolic to parabolic relaxation theory for quasilinear first order systems*, Preprint, 1998.

[135] Markowich, P. A. , Ringhofer, C. A. , and Schneiser, C. , *Semiconductor Equations*, Springer-Verlag, Wien, New York, 1990.

[136] Matsumura, A. and Nishida, T., *The iniitial value problem for the equations of motion of viscous and heat-conductive gases*, J. Math. Kyoto Univ. 20 (1980), 67–104.

[137] Matsumura, A. and Nishihara, K., *On the stability of travelling waves of a one-dimensional model system for compressible viscous gas*, Japan J. Appl. Math. 2(1985), 17-25.

[138] Metivier, G., *A nonlionear instability for 3x3 systems of conservation laws*, preprint, 1993.

[139] Michaelson, D., *Initial-boundary value problems for incomplete singular perturbations of hyperbolic systems*, J. Analyse Math. 53 (1089), 1–138.

[140] Moore, D. W., *The spontaneous apperance of a singularity of an evolving vortex sheets*, Proc. Roy. Soc. London Ser. A, 365 (1979), 105-119.

[141] Morawetz, C. S. *On steady transonic flow by compensated compactness*, Comm. Pure Appl. Math. Vol. XLVII (1994), 1–15.

[142] Natalini, R., *Convergence to equilibrium for the relaxation approximations of conservation laws*, Comm. Pure Appl. Math. 49 (1996), 795-823.

[143] Natalini, R. , *A discrete kinetic approximation of entropy solutions to multidimensional scalar conservation laws*, preprint, 1996.

[144] Necas, J. Ruzicka, M. and Sverak, V. *On self-similar solutions of the Navier-Stokes equations*, IMA preprint, 1995.

[145] Nishida, T., *Fluid dynamic limit of the nonlinear Boltzmann equation in the level of the compressible Euler equation*, Comm. Math. Phys. 61 (1978), 119-148.

[146] Oleinik, O. A., *The Prandtl system of equations in bounday layer theory*, Dokl. Akad. Nauk S.S,.S.R., 150 4(3) (1963), 583-586.

[147] Osher, S. and Shu, C. W. , *Efficient implementation of essentially non-oscillatory shock capyuring schemes*, J. Comp. Phys. 27 (1987), 1-31.

[148] Perthame, B., *Boltzmann type schemes for gas dynamics and the entropy property*, SIAM J. Num. 27 (1990), 1405-1421.

[149] Perthame, B. and Tadmor, E., *A kinetic equation with kinetic entropy functions for scalar conservation laws*, Comm. Math. phys. 136(1991), 501-517.

[150] Pullin, D. I. , *On self-similar flows containing two-branched vortex sheets*, in Mathematical Aspects of Vortex Dynamics, Caflisch ed. (1989), 97-106.

[151] Rauch, J. *BV estimates fail for most quasilinear hyperbolic systems in dimension greater than one*, Comm. Math. Phys. 106 (1986), 481-484.

[152] Sammartino, M. and Caflisch, R. E., *Zero viscosity limit for analytic solutions of the Navier-Stokes equations on a half space*, (I and II), Preprint, 1997.

[153] Schaeffer, D. G. and Shearer, M., *The classification of 2×2 systems of nonstrictly hyperbolic conservation laws, with application to oil recovery*, Comm. Pure Appl. Math. 40 (1987), 141–178.

[154] Schlichting, H., *Boundary-Layer Theory*, 7th Edition, McGraw-Hill, 1987.

[155] Serre, D. *La compacite par compensation pour les systemes hyperboliques nonlineaires de deux equations a une dimension d'espace*, J. de Math. Pures et Appl. 65 (1986), 423–468.

[156] Sideris, J. *Formation of singularities in three-dimensional compressible fluids*, Comm. Math. Phys. 101(1995), 475-487.

[157] Slemrod, M. , *Continuum dynamics of first order phase transitions*, In Phase Transitions and Material Instabilities in Solids, M. Gurtin (Ed.), Academic Press, London, 1984, 163–204.

[158] Slemrod, M. and Tzavaras., *Self-similar fluid dynamic limoits for the Broadwell system*, Arch. Rat. Mech. Anal. 122 (1993), 353-392.

[159] Smoller, J. A. *Shock Waves and Reaction-Diffusion Equations*, Berlin-Heidelberg-New York: Springer-Verlag, New York, 1984.

[160] Suffman, P. G. ,*Vortex Dynamics*, Cambridge Univ. Press, 1992.

[161] Szepessy, A., *High frequency asymptotics for 2D viscous shocks*, preprint, 1998.

[162] Szepessy, A. and Xin, Z. P., *Nonlinear stability of viscous shock waves*, Arch. Rat. Mech. anal. 122 (1993), 53-103.

[163] Szepessy, A. and Zumbrun, K. , *Stability of viscous rarefaction waves*, Arch. Rat. Mech. Anal. 102 (1995).

[164] Tan, D. and Xin, Z. P. , *Muti-branched self-similar vortex sheets for 2D Incompressible Euler Equations*, preprint, 1997.

[165] Tartar, L., *Compensated compactness and applications to partial differential equations*, Nonlinear Analysis and Mechanics: Heriot Watt Symposium IV (1979), 136–212.

[166] Temple, B., *Decay with a rate for noncompactly supported solutions of conservation laws*, Trans. AMS 298 (1986), 43–82.

[167] Temple, B. and Young, R. *Solutions to the Euler equations with large data*, Proc. 5th. Int'l. Conf. Hyperbolic Problems, Theory, Numerics, Applications, eds. by Glimm, etc. World Scientific, (1996), 258-267.

[168] Teng, Z. H. , *First order L^1-rate of convergence for relaxation approximations to conservation laws*, Comm. Pure Appl. Math. 51 (1998), 857–895.

[169] Tian, G. and Xin, Z. P., *Gradient estimation on Navier-Stokes Equations*, Comm. Geometrical Anal. Vol. 7 No. 2 (1999), 221–257.

[170] Tian, G. anf Xin, Z. P., *One-point singular solutions to the Navier-Stokes equations*, to appear in Top. Meth. Nonlinear Anal. .

[171] Trangenstein, and Collela, P., *A higher-order Godunov method for modelling finite deformation in elastic-plastic solids*, to appear.

[172] Tsai, T. P. , *On Leray's self-similar solutions of the Navier-Stokes equations satisfying local energy-estimates*, preprint, 1997.

[173] Ukhovskii,M. R. and Indovich, V. I., *Axially symmetric flows of ideal and viscous fluids filling the whole space*, J. Appl. Math. Mech. 32 (1968), 52-62.

[174] Vicenti, W. and Kruger, C. *Introduction to Physical Gas Dynamics*. Melbourne: Robert E. Krieger, 1982.

[175] Volpert, A. I. , *The space BV and quasilinear equations*, Math. USSR-Sb. 2 (1967), 225–267.

[176] Wang, W.C. and Xin, Z. P. *Asymptotic limit of the initial boundary value problems for conservation laws with relaxational extensions*, Comm. Pure Appl. Math. 51 (1998), 505-535.

[177] Wang, W.C. and Xin, Z. P., *On small mean free path limitof Broadwell model with discontinuous initial data, the centered rarefaction wave case*, J. Diff. Eqs. 150 (1998), 438–461.

[178] Whitham, J., *Linear and nonlinear waves*, Wiley (1978).

[179] Wieger, M., *Decay and satbility in L_p for strong solutions of Cauchy problems for the Navier-Stokes equations*, Lect. Notes Math. 1431, Berlin, Heidelberg, New York; Springer 1990, 95-99.

[180] Xin, Z. P. *On the blow-up of smooth solutions to the compressible Navier-Stokes equations with compact density*, Comm. Pure Appl. MAth. , 51 (1998), 229-240.

[181] Xin, Z. P. *Viscous boundary layers and their stability (I)*, J. Partial Diff. Eqs. , 11(1998), 97-124.

[182] Xin, Z. P. and Yanagisawa, T. , *Viscosity limit of the linearized Navier-Stokes equations for a compressible viscous fluid in the half plane*, Comm. Pure Appl. Math. Vol. LII (1999), 497–541.

[183] Xin, Z. P. , *On the nonlinear stability of Contact discontinuities*, Proc. 5th Int'l. Conf. Hyperbolic Problems: Theory, Numerics, Applications, edited by Glimm, etc., World Scientific, (1996) , 249-257.

[184] Xin, Z. P. , *Shock and boundary layer behavior of solutions to the Broadwell equations in the fluid-dynamic limit*, AMS/IP Studies in Advanced Mathematics, 3 (1997), 455-468.

[185] Xin, Z. P. , *Asymptotic stability of planar rarefaction waves for viscous conservation laws in several space dimensions*, Transac. AMS, 319 (1990), 805-820.

[186] Xin, Z. p. , *On the linearized stability of viscous shock profiles for systems of conservation laws*, J. Diff. Equations, 100 (1992), 119-136.

[187] Xin, Z. P. , *Asymptotic stability of rarefaction waves for 2×2 viscous hyperbolic conservation laws*, J. Diff. Eqs. 78 (1989), 191–219.

[188] Xin, Z. P. , *The fluid-dynamical limit of the Broadwell model of the nonlinear Boltzmann equation in the presence of shocks*, Comm. Pure Appl. Math. XLIV (1991), 679–713.

[189] Xin, Z. P. , *The fluid-dynamical limit of the Broadwell equation*, SIAM Vol. on Viscous and Numerical Approx. of Shock Waves, ed. M. Shearer (1991).

[190] Xin, Z. P. , *Zero dissipation limit to rarefaction waves for the one-dimensional Navier-Stokes equations for compressible isentropic gas*, Comm.. Pure Appl. Math. 86 (1993), 1499-1533.

[191] Xin, Z. P., *Some open problems in partial differential equations arising from fluid dynamics*, In Current Development of Mathematics (1997), Edited by Batt. etc.. International Press, 231-277.

[192] Xin, Z. P. and Xu, W. Q. , *Stiff well-posedness and asymptotic convergence for a class of linear relaxation systems in a quarter plane*, IMS Research Report IMS-99-04(041), 1999.

[193] Yang, T. *A functional integral approach to shock wave solutions of Euler equations with spherical symmetry*, Comm. Math. Phys. 171(1995), 607–638.

[194] Ying, H. C. and Xin, Z. P. , *The blowup of small data solutions for a class of quasilinear wave equations in three space dimensions*, preprint, 1999.

[195] Young, R., *An extension of Glimm's method to third order in wave interactions*, Ph.D. Thesis, Univ. of California, Davis (1991).

[196] Yu, S. H. , *Vanishing viscosity limits for systems of conservation laws*, preprint, 1998.

[197] Yudovitch, V. I. , *Nonstationary flow of an ideal incompressible liquid*, USSR Comput. Math. Math. Phys. 3(1963), 1407–1456.

[198] Zeng, Y., *Convergence to diffusion waves of solutions to nonlinear viscoelastic model with fading memory*, Comm. Math. Phys. 146 (1992), 585–609.

[199] Zeng, Y. *On gas dynamics in thermo-non-equilibrium*, preprint, 1999.

[200] Zhang, T. and Zheng, Y. , *Exact spiral solutions of the two-dimensional Euler equations*, preprint, 1998.

This page intentionally left blank

American
Mathematical
Society
www.ams.org

International
Press
www.intlpress.com

0-8218-1965-8

9 780821 819654

AMSIP/15