Teaching Mathematics in Colleges and Universities: Case Studies for Today's Classroom

Faculty Edition

Solomon Friedberg et al.
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The case studies in this volume are fictionalized accounts of common teaching situations. All characters in them are fictitious. Any similarities to specific individuals are purely coincidental.
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Teaching Mathematics in Colleges and Universities:
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Progress in mathematics frequently occurs first by studying particular examples and then by generalizing the patterns that have been observed into far-reaching theorems. Similarly, in teaching mathematics one often employs examples to motivate a general principle or to illustrate its use. This volume uses the same idea in the context of learning how to teach: By analyzing particular teaching situations, one can develop broadly applicable teaching skills useful for the professional mathematician. These teaching situations are the Case Studies of the title.

Just as a good mathematician seeks both to understand the details of a particular problem and to put it in a broader context, the examples presented here are chosen to offer a serious set of detailed teaching issues and to afford analysis from a broad perspective.

Each case raises a variety of pedagogical and communication issues that may be explored either individually or in a group facilitated by a faculty member. Teaching notes for such a facilitator are included for each Case in the Faculty Edition.

The methodology of Case Studies is widely used in areas such as business and law. The consideration of the mathematics cases presented here will help readers to develop teaching skills for their own classrooms.