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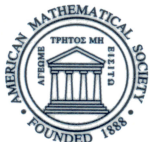
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Regional Conference Series in Mathematics

Number 1

Algebraic and Analytic Aspects of Operator Algebras

Irving Kaplansky



American Mathematical Society
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Irving Kaplansky

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PREFACE

These notes are a fairly accurate record of a series of ten lectures given at the University of Hawaii during the period June 9–13, 1969. The occasion was a Regional Conference sponsored by the Conference Board of the Mathematical Sciences, with the support of the National Science Foundation.

The central theme of the lectures was C^* -algebras, together with their numerous alphabetical progeny (W^* , AW^* , CCR , GCR). However, where appropriate, attention was paid to wider classes of Banach algebras, and the interplay between algebra and analysis received recognition by occasional forays into pure ring theory. It seemed particularly fitting to begin with a simple characterization of von Neumann regular rings which promises to be quite useful.

I learned a great deal from stimulating interchanges with people at the conference, including (but not limited to) Charles Akemann, Paul Halmos, Gerhard Kalisch, Bernard Russo, Lawrence Wallen, Joseph Wolf, Aubrey Wulfsohn, and Daniel Zelinsky. The University of Hawaii, ably represented by Adolf Mader, was an excellent host.

Chicago
July, 1969

Irving Kaplansky

Note for the second printing (December, 1979)

It is pleasant to report that virtually all the questions raised in this monograph have been answered. At least this is true if a question is settled when it turns out to be undecidable, for this was the fate of the problem concerning the continuity of homomorphisms on $C(X)$ (page 5).

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