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Lectures on the Edge-of-the-wedge Theorem

Walter Rudin

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a sequence of polynomials that converges uniformly to the given function f on the compact set

$$K = \overline{W^+} \cup \overline{W^-},$$

and hence converges also uniformly on the polynomial hull $\hat{K}$ of K. This gives the holomorphic extension of f to the interior Ω of $\hat{K}$. By embedding holomorphic discs whose boundaries lie in K one shows that $\Omega \supset E$.

($\hat{K}$ is, by definition, the set of all z in $\mathbb{C}^n$ at which

$$|P(z)| \leq \max\{|P(w)| : w \in K\}$$

for every polynomial P.)

Kolm and Nagel [24; p.91] seem to have been the first to consider 1-dimensional edges in $\mathbb{C}^n$. The lemma in [35] states essentially the same result:

Let E be a $\mathcal{C}^1$-curve in $\mathbb{C}$ which separates a disc $\Delta \subset \mathbb{C}$ into two regions, Δ^+ and Δ^-. Write points $z \in \mathbb{C}^n$ in the form $z = (z_1, z'), z' = (z_2, \cdots, z_n)$ and put

$$W^+ = \{z \in \mathbb{C}^n : z_1 \in \Delta^+, |z'| < \text{dist}(z_1, E \cap \Delta)\}$$
$$W^- = \{z \in \mathbb{C}^n : z_1 \in \Delta^-, |z'| < \text{dist}(z_1, E \cap \Delta)\}.$$

There is then an open set Ω in $\mathbb{C}^n$,

$$\Omega \supset W^+ \cup (E \cap \Delta) \cup W^-,$$

with the following property: If f is holomorphic in $W^+ \cup W^-$, and every derivative $D^\alpha f$ extends continuously to $W^+ \cup (E \cap \Delta) \cup W^-$, then f extends holomorphically to Ω.

Instead of having E in $\mathbb{C}$ one could also have E lie in any one-dimensional complex manifold.

Other results of this kind, some with generic edges of dimension greater than n, can be found in [16] and [34].

Edge-of-the-wedge theorems have been used to give simpler proofs of Fefferman's famous theorem [22] which asserts the $\mathcal{C}^\infty$-extendibility of biholomorphic maps between bounded strictly pseudoconvex domains in $\mathbb{C}^n$ with $\mathcal{C}^\infty$-boundaries. His original proof involved a difficult analysis of Bergman kernels and a detailed study of the geodesics with respect to the Bergman metric. For simplifications using reflections see [26], [38], [29]; also [25] and [21]. The simplest proof so far is probably the one by Forstnerič [23] which combines the edge-of-the wedge theorem (also used in [32]) with the n-variable version of the Julia-Caratheodory theorem [36].

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