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http://dx.doi.org/10.1090/cbms/022

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by

O. T. O'MEARA

Published for the

Conference Board of the Mathematical Sciences

by the

American Mathematical Society

Providence, Rhode Island

Expository Lectures
from the CBMS Regional Conference
held at Arizona State University
March 26–30, 1973

AMS (MOS) 1970 subject classifications: 15-02, 15A33, 15A36, 20-02, 20B25, 20D45, 20F55, 20H05, 20H20, 20H25; 10C30, 13-02, 15A36, 20F35, 20G15, 50D25, 50D30

Library of Congress Cataloging in Publication Data

O'Meara, O. Timothy, 1928–
Lectures on linear groups.
(Regional conference series in mathematics, no. 22)
"Expository lectures from the CBMS Regional Conference held at Arizona State University, March 26–30, 1973."
1. Matrix groups. 2. Matrices. 3. Automorphisms. 4. Transformations (Mathematics) I. Conference Board of the Mathematical Sciences.
II. Title. III. Series.
QA1.R33 no. 22 [QA171] 510′.8s [512′.2] 74-8773
ISBN 0-8218-1672-1

Reprinted with corrections, 1977
Reprinted 1988
Printed in the United States of America

The paper used in this journal is acid-free and falls within the guidelines established to ensure permanence and durability.

Contents

Preface

My goal in these lectures is the isomorphism theory of linear groups over integral domains as illustrated by the theorem

$$PSL_n(\mathfrak{o}) \cong PSL_{n_1}(\mathfrak{o}_1) \Longleftrightarrow n = n_1 \text{ and } \mathfrak{o} \cong \mathfrak{o}_1$$

for dimensions $\geqslant 3$. The theory that follows is typical of much of the research of the last decade on the isomorphisms of the classical groups over rings. I will start from scratch, assuming only basic facts from a first course in algebra. In particular, the classical theorem on the simplicity of $PSL_n(F)$ will be proved (in two different ways, as a matter of fact), and whatever is needed from projective geometry will be developed. Since our interest is in integral domains, we stay commutative throughout. In reorganizing the literature for these lectures I found it possible to extend the known theory from groups of linear transformations to groups of collinear transformations, and also to improve the isomorphism theory from dimensions $\geqslant 5$ to dimensions $\geqslant 3$. These new results are included in what follows.

These notes evolved from lectures at the California Institute of Technology during the spring of 1968, from ten survey lectures on classical and Chevalley groups at an NSF Regional Conference at Arizona State University in March 1973, and from lectures on linear groups at the University of Notre Dame in the fall of 1973.

I would like to express my thanks to Olga Taussky and Hans Zassenhaus for introducing me to the linear groups many years ago, to Warren Wong, Carl Riehm and Alex Hahn for countless discussions on the subject, and to Ronald Jacobowitz for a good conference.

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JANUARY 1974

O. T. O'MEARA

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