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Lectures on
Symplectic Manifolds

Alan Weinstein
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so  \( a(x) \) must be a constant multiple of  \( e^{iS(x)} \). The presence of this multiplicative constant is unavoidable, since the function  \( S(x) \) is determined by  \( L \) only up to an additive constant.

If we replace  \( dS \) by an arbitrary closed 1-form  \( \omega \) on  \( X \), the lagrangian submanifold  \( L = \omega(X) \) is quantizable just when the function  \( u(x) \) satisfying  \( du = \omega \) is globally defined modulo 2\( \pi \); this is the case when the integral of  \( \omega \) around every closed curve in  \( X \) is an integer multiple of 2\( \pi \), or, equivalently, when the class  \( [\omega/2\pi] \in H^1(X; \mathbb{R}) \) is in the image of  \( H^1(X; \mathbb{Z}) \). In this case, the function  \( e^{iu(x)} \) is globally defined and may be associated with  \( \omega(X) \).

We have now associated, to each quantizable lagrangian section  \( L = \omega(X) \subset T^*X \), a function on  \( X \). To get a 1/2-density on  \( X \), we should add one piece of information — a 1/2-density on  \( L \), which we can pull back to  \( X \) and multiply by the function. This suggests that the objects to be quantized in general should be pairs consisting of a lagrangian submanifold  \( L \) and a 1/2-density on  \( L \). Such a pair will be called a quasi-classical state (see [SL 2]). (In case  \( L \) is the graph of a symplectomorphism  \( f: P_1 \rightarrow P_2 \), it carries a natural 1/2-density induced from the symplectic form on  \( P_1 \) or  \( P_2 \).)

Remaining within cotangent bundles, we may ask next how to quantize a quasi-classical state  \((L, \delta)\) in  \( T^*X \) for which  \( L \) does not project diffeomorphically onto  \( X \). How, for example, should we quantize the fibre  \( x = 0 \) in  \( T^*\mathbb{R} \)? Any function on  \( T^*\mathbb{R} \times S^1 \) of the form  \( \varphi(x, \xi, 0) = e^{-i\theta} a(x) \) is already constant along the horizontal lifts of the fibre  \( x = 0 \) (as it is along the horizontal lifts of all fibres), so none is distinguished by that condition. Since the fibre  \( x = 0 \) lies only over the origin in  \( \mathbb{R} \), it is tempting to quantize it by a Dirac delta "function" supported at the origin. It turns out that argument by continuity leads to the same conclusion, as we shall now see.

Consider the line  \( L \) defined by  \( x = 0 \) in the  \((x, \xi)\)-plane, equipped with the 1/2-density  \( \rho = C\xi d\xi|^{1/2} \), where  \( C \) is a constant. It is the limit as  \( m \rightarrow 0 \) of the line  \( L_m \) defined by  \( x = m\xi \), equipped with the 1/2-density  \( \rho_m \) whose expression in the  \( \xi \)-coordinate is  \( C|\xi|^{1/2} \). Now  \( L_m \) is also defined by the equation  \( \xi = x/m \), so it is  \( dS(R) \), where  \( S(x) = x^2/(2m) \); in the  \( x \)-coordinate, our 1/2-density becomes

\[
C \left| d \left( \frac{x}{m} \right) \right|^{1/2} = \frac{C}{|m|^{1/2}} |dx|^{1/2}.
\]

By our quantization rule for sections of  \( T^*X \), we should associate with  \((L_m, \rho_m)\) the 1/2-density  \((C/|m|^{1/2})e^{ix^2/(2m)} |dx|^{1/2} \) on  \( X \). (This is not in  \( L^2 \); recall, however, that the objects associated with lagrangian submanifolds of  \((P, \Omega)\) are generally contained in some extension of the Hilbert space which quantizes  \((P, \Omega)\).) Since  \((L, \rho) = \lim_{m \rightarrow 0} (L_m, \rho_m)\), it is natural to try to quantize  \((L, \rho)\) by

\[
\lim_{m \rightarrow 0} \frac{C}{|m|^{1/2}} e^{ix^2/(2m)} |dx|^{1/2}.
\]

We cannot make sense out of the last limit in the space of  \( C^\infty \) 1/2-densities on  \( \mathbb{R} \), but the limit does exist in the following "weak" sense. If  \( u = u(x)|dx|^{1/2} \) is any  \( C^\infty \) 1/2-density on  \( \mathbb{R} \) with compact support and  \( u = u(x)|dx|^{1/2} \) is any  \( C^\infty \) 1/2-density on  \( \mathbb{R} \), we can form their product by
In this way, the space $D(|R|^{1/2})$ of $C^\infty$ 1/2-densities on $R$ is identified with a space of linear functionals on the space $D(|R|^{1/2})$ of compactly supported $C^\infty$ 1/2-densities. The full space of linear functionals on $D(|R|^{1/2})$, continuous with respect to a certain $C^\infty$ topology (see [S]), is denoted by $D'(|R|^{1/2})$. Since $D(|R|^{1/2})$ contains $E(|R|^{1/2})$ (as a dense subset), its elements are called generalized 1/2-densities, or 1/2-density-valued distributions, on $R$.

Now we may try to find the limit (10.1) in $D'(|R|^{1/2})$. To do this, we must evaluate

\[ \lim_{m \to 0} \int \frac{C}{|m|^{1/2}} e^{ix^2/2m} \tilde{v}(x) \, dx \]

where $v(x)$ is a compactly supported $C^\infty$ function on $R$. In fact, the principle of stationary phase tells us that the limit (10.2) exists if $m$ approaches 0 with a fixed sign; it is equal to

\[ C(2\pi)^{-1/2} e^{(i\pi/4)} \text{sgn } m \tilde{v}(0). \]

The functional $v(x) |dx|^{1/2}$ belongs to $D'(|R|^{1/2})$; it is called a Dirac delta functional at the origin and will be denoted by $\delta_0 |dx|^{1/2}$. Then we have, in $D'(|R|^{1/2})$,

\[ \lim_{m \to \pm 0} \frac{C}{|m|^{1/2}} e^{ix^2/2m} |dx|^{1/2} = C(2\pi)^{-1/2} e^{\pm i\pi/4} \delta_0 |dx|^{1/2}. \]

Thus, we are lead to quantize the quasi-classical state $(x = 0, C |d\xi|^{1/2})$ by the distribution (determined up to a factor of $i$) $C(2\pi)^{-1/2} e^{\pm i\pi/4} \delta_0 (|dx|^{1/2})$ on $X$.

We shall now describe a general construction which is motivated by this example. Let $V$ be a real vector space of dimension $n$. Any lagrangian subspace $L$ of $T^*V = V \times V^*$ which is transversal to the "vertical" space $(0) \times V^*$ (which we will denote simply by $V^*$) is the graph of a symmetric mapping $A_L: V \to V^*$; equivalently, $L = dS_L(V)$, where $S_L$ is the quadratic function $S_L(x) = \frac{1}{2} A_L(x)(x)$. (We remove the indeterminacy in $S_L$ by requiring $S_L(0) = 0$.) If $\rho$ is any 1/2-density on $L$, we may regard it as well as a 1/2-density on $V$ by pullback, and we quantize the pair $(L, \rho)$ by the 1/2-density $e^{iS_L(x)} \rho$ on $V$.

If $L$ is the vertical space, its quantization should be a delta functional supported at $0 \in V$. Again, the principle of stationary phase justifies this choice, but instead of working out the details of this we will pass immediately to the general problem of quantizing a pair $(L, \rho)$, where $L$ is an arbitrary lagrangian subspace of $T^*V$, and $\rho$ is a translation-invariant 1/2-density on $L$.

The lagrangian subspace $L$ projects onto a subspace $W_L \subseteq V$. We may guess that the quantization of $(L, \rho)$ should consist of distributions which are supported on $W_L$, but which distributions should they be? The subspace $F = W_L \oplus V^*$ is coisotropic in $V \oplus V^*$, and $F^\perp = 0 \oplus W_L^\perp$, where $W_L^\perp \subseteq V^*$ is the usual annihilator of $W_L$. Since $V^*/W_L^\perp \approx W_L^*$, the reduced symplectic space $F/F^\perp$ is naturally isomorphic to $W_L \oplus W_L^*$. The reduction $(L \cap F)/(L \cap F^\perp)$ of $L$ to $F/F^\perp \approx W_L \oplus W_L^*$ projects onto $W_L$, so it is the graph of a symmetric mapping $W_L \xrightarrow{A} W_L^*$.

This suggests that we quantize $(L, \rho)$ by the function $e^{(1/2)iA L(x)(x)}$ on $W_L$ times
some "delta functional" along $W_L$. In fact, some juggling of 1/2-densities and exact sequences (see [GU] or [W 7]) shows that there is a natural isomorphism $j_L$ between the space $|L|^{1/2}$ of 1/2-densities on the vector space $L$ and the space $\text{Hom}(|V|^{1/2}, |W_L|)$ of homomorphisms from 1/2-densities on $V$ to 1-densities on $W_L$. (In case $W_L = V$, $j_L$ is just the pull back isomorphism.) The latter space may be identified with a subspace of $D'(|V|^{1/2})$: Given $\sigma: |V|^{1/2} \to |W_L|$ and a "test-density" $u \in D(|V|^{1/2})$, we may apply $\sigma$ to it and restrict to $W_L$, obtaining an element of $D(|W_L|)$ which may be integrated over $W_L$ to give a complex number. Thus we are led to quantize $(L, \rho)$ by the functional

$$u \mapsto \int_{W_L} e^{(1/2)iA_L(x)(x)}(j_L\rho) \circ u,$$

which we will denote by $\delta(L, \rho)$.

The 1-dimensional case suggests that we should modify $\delta(L, \rho)$ by constant factors of $(2\pi)^{1/2}$ and $e^{i\pi/4}$. We shall now show that, with such modifications, the quantization of lagrangian subspaces becomes a continuous process.

We may study the process $(L, \rho) \to \delta(L, \rho)$ and, incidentally, define a differentiable structure on the lagrangian grassmannian $L(T^*V)$, by using a special covering of $L(T^*V)$. For each $K \in L(T^*V)$ which is transversal to $\{0\} \times V^*$ (which we denote simply by $V^*$), we define $U_K$ to be the set of $L \in L(T^*V)$ which are transversal to $K$. Given any $L \in L(T^*V)$, there is a $K$ which is transversal to both $L$ and $V^*$, so the $U_K$'s cover $L(T^*V)$.

Now we may identify $U_K$ with the space of quadratic forms on $V^*$. In fact, the lagrangian splitting $T^*V = V^* \oplus K$ induces an isomorphism of $K$ with $V^{**}$ (it is the negative of the isomorphism of $K$ with $V$ given by projection along $V^*$) as in Lecture 2, and hence an isomorphism of $T^*V$ with $V^* \oplus V^{**} = T^*(V^*)$. Now each $L$ in $U_K$ is identified with a lagrangian subspace $\alpha_K(L)$ of $T^*V$ which is transverse to $V^{**}$. By the earlier construction in this section, we then have the symmetric mapping $V^* \xrightarrow{A_{\alpha_K(L)}} V^{**}$ and the quadratic function $S_{\alpha_K(L)}(\xi) = \frac{1}{2}A_{\alpha_K(L)}(\xi)(\xi)$ on $V^*$. We will write $A(V^*|L|K)$ for $A_{\alpha_K(L)}$.

The mappings $U_K \xrightarrow{\varphi_K} \text{Sym}(V^*, V^{**})$ defined by $\varphi_K(L) = A(V^*|L|K)$, which are bijective, will be taken as the charts for $L(T^*V)$. To show that the charts give a differentiable structure, we must study the transition map on $U_{K_1} \cap U_{K_2}$. Writing $A(V|K|V^*)$ for the symmetric map from $V$ to $V^*$ of which $K$ is the graph, and identifying $V$ with $V^{**}$ in the usual way, we have the following lemma, whose proof we omit.

**Lemma.** (a) If $L \in U_{K_2}$, then $L$ lies in $U_{K_1}$ if and only if the operator

$$I - A(V^*|L|K_2)[A(V|K_2|V^*) - A(V|K_1|V^*)]$$

is invertible.

(b) If $L \in U_{K_1} \cap U_{K_2}$, then

$$\varphi_{K_1}(L) = (I - \varphi_{K_2}(L)[A(V|K_2|V^*) - A(V|K_1|V^*)])^{-1}\varphi_{K_2}(L).$$

This lemma implies immediately that the charts $(U_K, \varphi_K)$ define a differentiable structure on $L(T^*V)$.

We return now to the quantization of lagrangian subspaces. The correspondence
(L, ρ) → δ(L, ρ) is clearly continuous for those L which are transversal to the vertical. We look next at those L which are transversal to the “horizontal”, i.e. L ∈ ℓV where we write V for V ⊕ {0}. The trick is to reduce this to the first case by using the Fourier transform, which is the analytic analogue of interchanging V and V*.

Suppose for the moment that L is transversal to both the horizontal and the vertical. It will help to use coordinates in what follows, so let (x₁, ..., xₙ) be coordinates on V and (ξ₁, ..., ξₙ) the dual coordinates on V*. We write |dx|¹/² for |dx₁ ∧ ⋯ ∧ dxₙ|¹/² and the same for the ξ’s. L is described by the equation ξ = Ax, where A = A(V|L|V*) is symmetric and invertible (since L is transversal to the horizontal) and ρ = c|dx|¹/² for some constant c. δ(L, ρ) is then cε(1/2)|A(x)(x)|dx|¹/², and its Fourier transform is

\[ Fδ(L, ρ) = \left\{ e^{-it, x} e^{i(\pi/4)A(x)(x)} dx \right\} \mid \xi \mid^{1/2}. \]

(The “natural” Fourier transform on 1/2-densities involves the canonical 1/2-density |dx|¹/²|dξ|¹/² on T*V.) This Fourier transform is computed, for instance, on pp. 144–145 of [HO]. It is

\[ Fδ(L, ρ) = c(2\pi)^{n/2} e^{i(\pi/4)sgn A} e^{-(\pi/2)A^{-1}(t)(t)} \mid \det A \mid^{-1/2} \mid dξ \mid^{1/2}, \]

where sgn A is the signature of the symmetric form A. Now we may interpret the formula for Fδ(L, ρ) in terms of the coordinate φ on ℓV. In fact, B = A(V*|L|V) is the negative of the inverse of A = A(V|L|V*), and |det A|⁻¹/²|dξ|¹/² = |dx|¹/² on L, so c|det A|⁻¹/²|dξ|¹/² is just the pullback \( \overline{\rho} \) of \( \rho \) to V* by the projection along V, so we may write

\[ (10.3) \quad Fδ(L, ρ) = (2\pi)^{n/2} e^{-i(\pi/4)sgn B} e^{i(\pi/2)B(t)(t)} \overline{ρ}. \]

If L is not transversal to the vertical, we get a similar result. For instance, if L = V* and ρ = c|dξ|¹/², then δ(V*, ρ) = cδ₀|dx|¹/², and

\[ Fδ(V*, ρ) = c|dξ|^{1/2} = \overline{ρ}, \]

which we can write in the form (10.3) if we remove the factor (2\pi)^{n/2}, since B = A(V*|V*|V) is zero in this case.

A similar computation shows that, for general L ∈ ℓV,

\[ (10.4) \quad Fδ(L, ρ) = (2\pi)^{-(1/2)\dim W_L} e^{-i(\pi/4)sgn B} e^{i(\pi/2)B(t)(t)} \overline{ρ} \]

where B = A(V*|L|V) and \( \overline{ρ} \) is the pullback of \( ρ \) to V*.

We can see now how to “correct” δ(L, ρ). The signature of B is congruent mod 2 to the rank of B, which is in turn equal to the dimension of W_L. It follows from (10.4) that, except for “multiplicative jumps” of powers of eπ/2 = i when the signature of B changes, the map

\[ (L, ρ) \mapsto (2\pi)^{-(1/2)\dim W_L} e^{i(\pi/4)\dim W_L} \delta(L, ρ) \]

is continuous for L ∈ ℓV. (The jumps occur when L fails to be transversal to V*.)

To take into account the jumps, we may associate to (L, ρ) the 4-tuple

\[ ε(L, ρ) = \{ (2\pi)^{-(1/2)\dim W_L} e^{i(\pi/4)\dim W_L} i^k \delta(L, ρ) \mid k = 0, 1, 2, 3 \} \]
of distributions, which does depend continuously on \((L, \rho)\).

Finally, we must see what happens on \(U_K\) for \(K\) other than \(V\). It turns out that the map \((L, \rho) \mapsto \sigma(L, \rho)\) is still continuous — instead of just the Fourier transform, one must consider

\[
F(e^{-(i/2)A(V^*|L|K)^{-1}(x)}\delta(L, \rho)).
\]

Now consider the set \(M(T^*V) = \{(L, \delta(L, \rho)) | \rho \in \{|L|^{1/2}\} \subseteq L(T^*V) \times D'(|V|^{1/2})\}
\), with the projection \(M(T^*V) \twoheadrightarrow L(T^*V)\) given by \((L, \delta(L, \rho)) \mapsto L\). Our calculations show that \(M(T^*V)\) is a complex line bundle over \(L(T^*V)\) whose structure group reduces to the discrete group of multiplications by \(\{1, i, -1, -i\}\). Analysis of the jumps when \(L\) is not transversal to \(V\) shows that this bundle is exactly the Maslov line bundle used in the theory of Fourier integral operators. We have seen, therefore, an "analytic" realization of the Maslov bundle arising from quantization theory.

What structures on \(T^*V\) did we use to construct the Maslov bundle? Besides the symplectic structure and linear structure, we used the polarization given by the vertical space \(V^*\), but the horizontal space \(V\) played no essential role. In general, if we have a symplectic vector space \(E\) with a distinguished linear real polarization \(\pi\), we may prequantize \(E\) as a symplectic manifold and consider the distribution space \(D'(E, \pi)\) in which the Hilbert space obtained by geometric quantization sits as a dense subspace. (The choice of a horizontal space enables one to identify \(D'(E, \pi)\) with \(D'(|E/\pi|^{1/2})\).) In the bundle \(L(E) \times D'(E, \pi)\) over \(L(E)\), there is then a distinguished "Maslov bundle" \(M(E, \pi)\). This construction can be applied fibre by fibre, if we have a symplectic vector bundle \(E \to \Omega\) with two real polarizations to produce a line bundle over \(B\). If \(E = TL(T^*X)\), where \(L \subseteq T^*X\) is a lagrangian submanifold and the polarizations are given by \(TL\) and \(T_L\) (fibres), one recovers the Maslov bundle over \(L\) used in [HÖ]. Its holonomy is the mod 4 reduction of the Maslov class which we defined in Lecture 6.

This analytic construction of the Maslov bundle puts the theory of Fourier integral operators in a new perspective and suggests an extension of Hörmander's symbol construction to arbitrary distributions (see [WE 6] and [WE 7]). On the other hand, the theory of Fourier integral operators itself provides a means for quantizing certain lagrangian submanifolds. We refer the reader to [D] and [HÖ] for expositions of this theory, mentioning only that the phase functions of Lecture 6 play an important role.
References


1References were updated for the 1979 printing.


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