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LOUIS AUSLANDER

LECTURE NOTES ON NIL-THETA FUNCTIONS

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by

Louis Auslander

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Introduction

The material in these notes represents joint work with Richard Tolimieri and was much influenced by my previous joint work with Jonathan Brezin. In addition, some of the concepts were developed in a joint effort of myself, Tolimieri and Barry Kolb that was announced in [7]. In presenting the material, I have tried to lay a careful foundation, and I have stressed low-dimensional examples and special computations even when I later prove general results by general techniques. Also, the last two sections are minimally developed with the interested reader being urged to consult Tolimieri [11] for a complete treatment.

The relation of these Notes to the classical literature should be self-evident and the results in [3] will make this more specific for the interested reader. However, these notes do something that may not be so evident. In three important works [14], [12], [13] A. Weil presents a proof of the Plancherel Theorem, a new treatment of Abelian varieties and what we now call the Weil-Brezin map. That all these are inter-related is by no means apparent. In these notes they all become united in the study of nil-theta functions.

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