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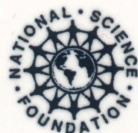
Number 38

Isolated Invariant Sets and the Morse Index

Charles Conley



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Published for the
Conference Board of the Mathematical Sciences
by the
American Mathematical Society
Providence, Rhode Island
with support from the
National Science Foundation

Expository Lectures
from the CBMS Regional Conference
held at the University of Colorado
May 31–June 4, 1976

1991 *Mathematics Subject Classification*. Primary 34C35;
Secondary 54H20.

Library of Congress Cataloging-in-Publication Data

Conley, Charles C., 1933–

Isolated invariant sets and the Morse index.

p. cm. — (Regional conference series in mathematics, ISSN 0160-7642; no. 38)

“Expository lectures from the CBMS regional conference held at the University of Colorado, May 31–June 4, 1976.”

Bibliography: p.

1. Differential equations. 2. Differentiable dynamical systems. 3. Topological dynamics.

I. Conference Board of the Mathematical Sciences. II. Title. III. Title: Morse index.

IV. Series.

QA1.R33 no. 38 [QA372] 510'.8s [515'.35] 78-1577

ISBN 0-8218-1688-8

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10 9 8 7 6 5 4 3 01 00 99 98 97 96

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