Topological Degree Methods in Nonlinear Boundary Value Problems

Jean Mawhin

TOPOLOGICAL DEGREE METHODS
IN NONLINEAR BOUNDARY VALUE PROBLEMS

Other Monographs in this Series

Conference Board of the Mathematical Sciences
REGIONAL CONFERENCE SERIES IN MATHEMATICS

supported by the
National Science Foundation

Number 40

TOPOLOGICAL DEGREE METHODS
IN NONLINEAR BOUNDARY VALUE PROBLEMS

by
J. MAWHIN

Expository Lectures
from the CBMS Regional Conference
held at Harvey Mudd College
June 9–15, 1977

2000 *Mathematics Subject Classification*. Primary 47–XX; Secondary 34–XX, 55–XX, 58–XX.

Contents

This page intentionally left blank

References

1. L. Aguinaldo and K. Schmitt, *On the boundary value problem $u'' + u = \alpha u^- + p(t)$, $u(0) = u(\pi) = 0$*, Proc. Amer. Math. Soc. **68** (1978), 64–68.

2. K. Ako, *Subfunctions for ordinary differential equations*. I, J. Fac. Sci. Univ. Tokyo (1) **12** (1965), 17–43; II, Funkcial. Ekvac. **10** (1967), 145–162; III, ibid. **11** (1968), 111–129; IV, ibid. **11** (1968), 185–195; V, ibid. **12** (1969), 239–249; VI, J. Fac. Sci. Univ. Tokyo (1) **16** (1969), 149–156.

3. P. Alexandroff and H. Hopf, *Topologie*, Springer-Verlag, Berlin, 1935.

4. M. Altman, *A fixed point theorem for completely continuous operators in Banach spaces*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **3** (1955), 490–513.

5. H. Amann, *Lectures on some fixed point theorems*, Monografias de Matematica, IMPA, Rio de Janeiro (to appear).

6. ———, *Fixed point equations and nonlinear eigenvalue problems in ordered Banach spaces*, SIAM Rev. **18** (1976), 620–709.

7. H. Amann and S. A. Weiss, *On the uniqueness of the topological degree*, Math. Z. **130** (1973), 39–54.

8. R. Ansorge and B. Mehri, *A generalization of a theorem of Reissig for a certain non-autonomous differential equation*, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. **61** (1976), 202–210.

9. A. B. Antonevich and V. B. Ryvkin, *Normal solvability in periodic solutions of a linear differential equation with a deviating argument*, Differential Equations **10** (1974), 1041–1046.

10. H. A. Antosiewicz, *A fixed point theorem and the existence of periodic solutions*, in Proc. Fifth Internat. Conf. Nonlinear Oscillations, vol. 1, Izdat Akad. Nauk. Ukrain. SSSR, Kiev, 1971, pp. 40–44.

11. ———, *A general approach to linear problems for nonlinear ordinary differential equations*, in Ordinary Differential Equations, L. Weiss, ed., Academic Press, New York, 1971, pp. 1–9.

12. ———, *Fixed point theorems and ordinary differential equations*, in Studies in Ordinary Differential Equations, J. K. Hale, ed., Math. Assoc. Amer., Washington, D. C., 1977, pp. 169–200.

13. ———, *A new view on constrained problems*, In Nonlinear Systems and Applications, V. Lakshmikantham, ed., Academic Press, New York, 1977, pp. 15–19.

14. ———, *Some remarks on the existence of periodic solutions*, in Mélanges Th. Vogel, B. Rybak, P. Janssens and M. Jessel, ed., Presses Univ. Bruxelles, Bruxelles, 1978, pp. 7–17.

15. N. B. Azbelev, *On linear boundary value problems for functional differential equations*, Differencial'nye Uravnenija **10** (1974), 579–584.

16. N. Basile and M. Mininni, *About the solvability of some nonlinear functional equations in locally convex spaces*, Rend. Accad. Sci. Fis. Mat. Napoli (4) **44** (1977), 373–385.

17. ——, *A nonlinear spectral approach to Landesman-Lazer type problems* (to appear).

18. J. W. Bebernes, *A sinple alternative problem for finding periodic solutions of second order ordinary differential systems,* Proc. Amer. Math. Soc. **42** (1974), 121–127.

19. J. W. Bebernes and M. Martelli, *Periodic solutions for Liénard systems,* in Equadiff 78, Convegno Internaz. Equaz. Differ. Ordin. ed Equaz. Funzionali, R. Conti, G. Sestini and G. Villari, ed., Univ. Firenze, Firenze, 1978, pp. 535–546.

20. M. Berger, *On periodic solutions of second order Hamiltonian systems,* J. Math. Anal. Appl. **29** (1970), 512–522.

21. ——, *Periodic solutions of second order dynamical systems and isoperimetric variational problems,* Amer. J. Math. **93** (1971), 1–10.

22. ——, *On a family of periodic solutions of Hamiltonian systems,* J. Differential Equations **10** (1971), 17–26.

23. ——, *Nonlinearity and functional analysis. Lectures on nonlinear problems in mathematical analysis,* Academic Press, New York, 1977.

24. M. Berger and M. Berger, *Perspectives in nonlinearity,* Benjamin, New York, 1968.

25. S. R. Bernfeld, G. Ladde and V. Lakshmikantham, *Nonlinear boundary value problems and several Lyapunov functions for functional differential equations,* Boll. Un. Mat. Ital. **10** (1974), 602–613.

26. S. Bernstein, *Sur la généralisation du problème de Dirichlet,* Math. Ann. **62** (1906), 253–271; **69** (1910), 82–136.

27. N. A. Bobylev, *A two-point boundary value problem,* Differencial'nye Uravnenija **11** (1975), 2121–2133; 2298.

28. P. Bohl, *Uber die Bewegung eines mechanicsches Systems in der Nähe einer Gleichgewichtslage,* J. Reine Angew. Math. **127** (1904), 179–276.

29. K. Borsuk, *Drei Sätze über die n-dimensional Euklidische Sphäre,* Fund. Math. **20** (1933), 177–190.

30. T. T. Bowman, *Perturbations of Liénard systems with symmetries,* in Dynamical Systems, A. R. Bednarek, L. Cesari, ed., Academic Press, New York, 1977, pp. 1–10.

31. ——, *Periodic solutions of Liénard systems with symmetries,* Nonlinear Analysis, Theory, Methods and Applications **2** (1978), 457–464.

32. H. Brezis and L. Nirenberg, *Characterizations of the ranges of some nonlinear operators and applications to boundary value problems,* Ann. Scuola Norm. Sup. Pisa (4) **5** (1978), 225–326.

33. L. E. J. Brouwer, *Uber Abbildung von Mannigfaltigkeiten,* Math. Ann. **71** (1912), 97–115.

34. F. E. Browder, *On a generalization of the Schauder fixed point theorem,* Duke Math. J. **26** (1959), 291–303.

35. ——, *Topology and non-linear functional equations,* Studia Math. **31** (1968), 189–204.

36. ——, *Nonlinear operators and nonlinear equations of evolution in Banach spaces,* Proc. Sympos. Pure Math., vol. 18, part 2, Amer. Math. Soc., Providence, R. I., 1976.

37. F. E. Browder and W. Petryshyn, *Approximation methods and the generalized topological degree for nonlinear mappings in Banach spaces,* J. Functional Analysis **3** (1968), 217–245.

38. A. Cambini and A. M. Micheletti, *Soluzioni periodiche di sistemi nonlineari conservativi,* Boll. Un. Mat. Ital. **10** (1974), 713–723.

39. L. Cesari, *The alternative method in nonlinear oscillations,* in Studies in Ordinary Differential Equations, J. K. Hale, ed., Math. Assoc. America, Washington, D. C., 1977, pp. 201–235.

40. ———, *Qualitative conditions for nonlinear oscillations,* in VII Internat. Konf. Nichtlin. Schwingungen, Berlin, I.1, Akademie Verlag Berlin, 1977, pp. 143–154.

41. ———, *Functional analysis, nonlinear differential equations and the alternative method,* in Nonlinear Functional Analysis and Differential Equations, L. Cesari, R. Kannan and J. D. Schuur, ed., Dekker, New York, 1977, pp. 1–197.

42. L. Cesari and R. Kannan, *Solutions in the large of Liénard equations with forcing terms,* Ann. Mat. Pura Appl. **121** (1976), 101–124.

43. S. H. Chang, *Periodic solutions of certain n-th order nonlinear differential equations,* Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. **57** (1974), 519–524.

44. ———, *Existence of periodic solutions to second order nonlinear equations,* J. Math. Anal. Appl. **52** (1975), 255–259.

45. ———, *Periodic solutions of certain differential equations with quasibounded nonlinearities,* J. Math. Anal. Appl. **56** (1976), 165–171.

46. D. C. Clark, *On periodic solutions of autonomous Hamiltonian systems of ordinary differential equations,* Proc. Amer. Math. Soc. **39** (1973), 579–584.

47. R. H. Cole, *Theory of ordinary differential equations,* Century, Crofts, New York, 1968.

48. R. Conti, *Recent trends in the theory of boundary value problems for ordinary differential equations,* Boll. Un. Mat. Ital. (3) **22** (1967), 135–178.

49. K. L. Cooke, *Some recent work on functional-differential equations,* in Proc. U. S.-Japan Seminar on Differential and Functional Equations, Benjamin, New York, 1967, pp. 27–47.

50. W. A. Coppel, *Stability and asymptotic behavior of differential equations,* Heath, Boston, Mass., 1965.

51. C. Corduneanu, *Systèmes différentiels admettant des solutions bornées,* C. R. Acad. Sci. Paris **245** (1957), 21–24.

52. J. Cronin, *Fixed points and topological degree in nonlinear analysis,* Math. Surveys, no. 11, Amer. Math. Soc., Providence, R. I., 1964.

53. ———, *Periodic solutions of nonautonomous equations,* Boll. Un. Mat. Ital. **6** (1972), 45–54.

54. ———, *Equations with bounded nonlinearities,* J. Differential Equations **14** (1973), 581–596.

55. E. N. Dancer, *On the structure of solutions of non-linear eigenvalue problems,* Indiana Univ. Math. J. **23** (1974), 1069–1074.

56. E. N. Dancer, *Boundary value problems for ordinary differential equations on infinite intervals,* Proc. London Math. Soc. **30** (1975), 76–94.

57. ——, *Boundary value problems for weakly nonlinear ordinary differential equations,* Bull. Austral. Math. Soc. **15** (1976), 321–328.

58. ——, *On the range of certain damped nonlinear differential equations* (to appear).

59. D. G. De Figueiredo and J. P. Gossez, *Nonlinear perturbations of a linear elliptic problem near its first eigenvalue,* J. Differential Equations (to appear).

60. K. Deimling, *Nichtlineare Gleichungen und Abbildungsgrad,* Springer-Verlag, Berlin, 1974.

61. R. I. De Vries, *Periodic solutions of differential systems of Liénard and Rayleigh type,* in Dynamical Systems, A. R. Bednarek and L. Cesari, ed., Academic Press, New York, 1977, pp. 405–409.

62. S. Eilenberg and N. Steenrod, *Foundations of algebraic topology,* Princeton Univ. Press, Princeton, N. J., 1952.

63. K. D. Elworthy, and A. J. Tromba, *Degree theory on Banach manifolds,* in Proc. Sympos. Pure Math., vol. 18, part 1, 1970, Amer. Math. Soc., Providence, R. I., 1970, pp. 86–94.

64. J. O. C. Ezeilo, *On the existence of periodic solutions of certain third order differential equations,* Proc. Cambridge Philos. Soc. **56** (1960), 381–389.

65. R. Faure, *Sur l'application d'un théorème de point fixe à l'existence de solutions périodiques,* C. R. Acad. Sci. Paris Sér. A **282** (1976), 1295–1298.

66. ——, *Solutions périodiques d'équations admettant des pôles: étude par la méthode de Leray-Schauder et par un théorème de point fixe,* C. R. Acad. Sci. Paris Sér. A **283** (1976), 481–484.

67. R. Fennel and P. Waltman, *Boundary value problems for functional differential equations,* Bull. Amer. Math. Soc. **75** (1969), 487–489.

68. ——, *A boundary value problem for a system of nonlinear functional differential equations,* J. Math. Anal. Appl. **26** (1969), 447–453.

69. C. Fenske, *Analytische Theorie des Abbildungsgrades für Abbildungen in Banachraümen,* Math. Nachr. **48** (1971), 279–290.

70. ——, *A bifurcation theorem of Rabinowitz type for Fredholm mappings,* Bericht. Gesselsch. Math. Datenverarbeiting, no. 103, 1975, Bonn, p. 59.

71. W. T. Ford, *On the first boundary value problem for* $(h(x, x', t))' = f(x, x', t)$, Proc. Amer. Math. Soc. **35** (1972), 491–498.

72. I. Fredholm, *Sur une classe d'équations fonctionnelles,* Acta Math. **26** (1902), 1561–1564; **27** (1903), 365–390.

73. K. O. Friedrichs, *Special topics in analysis,* New York Univ. Lecture Notes, New York, 1953–1954.

74. S. Fucik, *Periodic solutions of generalized Liénard equations with forcing terms,* in Colloq. Math. Soc. János Bolyai, no. 15, Differential Equations, 1975, pp. 155–169.

75. ——, *Ranges of nonlinear operators,* 5 volumes, Univ. Carolinae Pragensis, Praha, 1978.

76. S. Fucik, O. John and A. Kufner, *Function spaces,* Academia, Praha, 1977.

77. S. Fucik, J. Necas, J. Soucek and Vl. Soucek, *Spectral analysis of nonlinear operators,* Lecture Notes in Math., vol. 346, Springer-Verlag, Berlin and New York, 1973.

78. M. Furi, M. Martelli and A. Vignoli, *Contributions to the spectral theory for nonlinear operators in Banach spaces,* Ann. Mat. Pura Appl. (to appear).

79. R. E. Gaines, *Existence of solutions to Hamiltonian dynamical systems of optimal growth,* J. Econom. Theory **12** (1976), 114–130.

80. ———, *Existence and the Cass-Shell stability condition for Hamiltonian systems of optimal growth,* J. Econom. Theory **15** (1977), 16–25.

81. ———, *Existence of solutions to Hamiltonian dynamical systems of economic growth with Marxian saving hypothesis,* in Equadiff 78, Convegno Internaz. Equaz. Diff. Oı in. ed Equaz. Funzionali, R. Conti, G. Sestini and G. Villari, ed., Univ. di Firenze, Firenze, 1978, pp. 191–212.

82. R. E. Gaines and C. P. Gupta, *Ordinary differential systems of optimal control type with monotone reversed nonlinearities,* J. Math. Anal. Appl. **64** (1978), 494–504.

83. R. E. Gaines and J. Mawhin, *Ordinary differential equations with nonlinear boundary conditions,* J. Differential Equations **26** (1977), 200–222.

84. ———, *Coincidence degree and nonlinear differential equations,* Lecture Notes in Math., vol. 568, Springer-Verlag, Berlin and New York, 1977.

85. I. C. Gokhberg and M. G. Krein, *The basic propositions on defect numbers, root numbers and indices of linear operators,* Amer. Math. Soc. Transl. (2) **13** (1960), 185–264.

86. J. P. Gossez, *Existence of periodic solutions for some first order ordinary differential equations,* in Equadiff 78, Convegno Internaz. Equaz. Diff. Ordin. ed Equaz. Funzionali, R. Conti, G. Sestini and G. Villari, ed., Univ. di Firenze, Firenze, 1978, pp. 361–379.

87. ———, *Periodic solutions for some nonlinear equations at resonance* (to appear).

88. D. Graffi, *Quelques limitations pour les solutions périodiques d'équations différentielles non linéaires,* in Mélanges Th. Vogel, B. Rybak, P. Janssens and M. Jessel, ed., Presses Univ. de Bruxelles, Bruxelles, 1978, pp. 153–166.

89. A. Granas, *Sur la méthode de continuité de Poincaré,* C. R. Acad. Sci. Paris Sér. A **282** (1976), 983–985.

90. V. Guillemin and A. Pollack, *Differential topology,* Prentice-Hall, Englewood Cliffs, N. J., 1974.

91. G. B. Gustafson, *An integral equation for perturbed nonlinear functional differential equations, with applications to periodic solutions and nonlinear boundary value problems,* J. Reine Angew. Math. **271** (1974), 35–62.

92. G. B. Gustafson and K. Schmitt, *A note on periodic solutions for delay-differential systems,* Proc. Amer. Math. Soc. **42** (1974), 161–166.

93. J. Hadamard, *Sur quelques applications de l'indice de Kronecker,* in Introduction à la Théorie des Fonctions d'une Variable, par J. Tannery, vol. II, Hermann, Paris, 1910, pp. 875–915.

94. A. Halanay, *A boundary value problem for linear systems with time lags,* J. Differential Equations **2** (1966), 47–56.

95. J. K. Hale, *Ordinary differential equations,* Wiley, New York, 1969.

96. J. K. Hale, *Applications of alternative problems,* Brown University Lecture Notes 71–1, Providence, R. I., 1971.

97. ——, *Oscillations in neutral functional differential equations,* in Nonlinear Mechanics, CIME, Cremonese, Roma, 1973, pp. 99–111.

98. ——, *α-contractions and differential equations,* in Equations différentielles et fonctionnelles non linéaires, P. Janssens, J. Mawhin and N. Rouche, ed., Hermann, Paris, 1973, pp. 15–42.

99. ——, *Theory of functional differential equations,* Springer, New York, 1977.

100. P. Hartman, *Ordinary differential equations,* Wiley, New York, 1964.

101. E. Heinz, *An elementary theory of the degree of mappings in n-dimensional spaces,* J. Math. Mech. **8** (1959), 231–247.

102. D. Henry, *The adjoint of a linear functional differential equation and boundary value problems,* J. Differential Equations **9** (1971), 55–66.

103. G. Hetzer, *Some remarks on ϕ_+-operators and on the coincidence degree for a Fredholm equation with noncompact nonlinear perturbation,* Ann. Soc. Sci. Bruxelles Sér. I **89** (1975), 497–508.

104. ——, *Bifurcation theorems of Rabinowitz type,* Nonlinear Analysis, Theory, Methods, Appl. **1** (1977), 471–479.

105. ——, *Boundary value-problems for retarded functional differential equations* (to appear).

106. ——, *A note on periodic solutions of second order systems of nonlinear autonomous differential equations* (to appear).

107. ——, *Alternativ- und Verzweigungsprobleme bei Koinzidenzgleichungen mit Anwendungen auf Randwertprobleme bei neutralen Funktionaldifferential- und elliptischen Differentialgleichungen,* Habilitationschrift, Aachen, 1978.

108. M. W. Hirsch, *Differential topology,* Springer-Verlag, New York, 1976.

109. E. Hopf, *Abzweigung einen periodischen Lösung von einer stationären Lösung eines Differentialsystem,* Ber. Ver. Sächs. Akad. Wiss. Leipzig, Math. Natur. Kl. **95** (1942), 3–22.

110. D. K. Hughes, *The adjoint of a linear functional differential operator,* J. Differential Equations **18** (1975), 11–17.

111. J. Ize, *Bifurcation theory for Fredholm operators,* Mem. Amer. Math. Soc. No. **174** (1976).

112. G. S. Jones, *Asymptotic fixed point theorems and periodic systems of functional differential equations,* Contributions Differential Equations **2** (1963), 385–405.

113. E. W. Kamen, *An operator theory of linear functional differential equations,* J. Differential Equations **27** (1978), 274–297.

114. R. Kannan, *Nonlinear perturbations at resonance,* in Dynamical Systems: An International Symposium, L. Cesari, J. K. Hale and J. P. LaSalle, ed., Vol. 2, Academic Press, New York, 1976, pp. 67–71.

115. ——, *Perturbation methods for nonlinear problems at resonance,* in Nonlinear Functional Analysis and Differential Equations, L. Cesari, R. Kannan and J. D. Schuur, ed., Dekker, New York, 1977, pp. 209–225.

116. R. Kannan and J. Locker, *Nonlinear boundary value problems and operators TT**, J. Differential Equations **28** (1978), 60–103.

117. R. Kannan and J. Schuur, *Nonlinear boundary value problems and Orlicz spaces*, Ann. Mat. Pura Appl. **113** (1977), 245–254.

118. J. L. Kazdan and F. W. Warner, *Remarks on some quasilinear elliptic equations*, Comm. Pure Appl. Math. **28** (1975), 567–597.

119. W. G. Kelley, *Second order systems with nonlinear boundary conditions*, Proc. Amer. Math. Soc. **62** (1977), 287–292.

120. ———, *A geometric method of studying two point boundary value problems for second order systems*, Rocky Mountain J. Math. **7** (1977), 251–263.

121. M. Konecky, *Remarks on periodic solvability of nonlinear ordinary differential equations*, Comment. Math. Univ. Carolinae **18** (1977), 547–562.

122. S. N. Krachkovskiĭ and A. S. Dikanskii, *Fredholm operators and their generalization*, in Mathematical Analysis 1968, Akad. Nauk SSSR, Moscow, 1969, pp. 39–71. (Russian)

123. A. M. Krall, *The development of general differential and general differential-boundary systems*, Rocky Mountain J. Math. **5** (1975), 493–542.

124. M. A. Krasnosel'skiĭ, *On a fixed point principle for completely continuous operators in function spaces*, Dokl. Akad. Nauk SSSR **74** (1950), 13–15. (Russian)

125. ———, *On the application of methods of nonlinear functional analysis to some problems about periodic solutions of the equations of nonlinear mechanics*, Dokl. Akad. Nauk SSSR **111** (1956), 283–286.

126. ———, *Topological methods in the theory of nonlinear integral equations*, Pergamon, Oxford, 1963.

127. ———, *An alternative principle for establishing the existence of periodic solutions of differential equations with a lagging argument*, Soviet Math. Dokl. **4** (1963), 1412–1415.

128. ———, *Positive solutions of operator equations*, Noordhoff, Groningen, 1964.

129. ———, *La théorie des solutions périodiques d'équations différentielles non autonomes*, Russian Math. Surveys **21** (1966), 53–74.

130. ———, *The operator of translation along trajectories of ordinary differential equations*, Transl. Math. Monographs, vol. 19, Amer. Math. Soc., Providence, R. I., 1968.

131. M. A. Krasnosel'skiĭ and E. A. Lifschitz, *Duality principles for boundary value problems*, Soviet Math. Dokl. **8** (1967), 1236–1239.

132. ———, *Some criteria for the existence of periodic oscillations in nonlinear systems*, Automat. Remote Control **9** (1973), 1374–1377.

133. M. A. Krasnosel'skiĭ and A. I. Perov, *On a certain principle for the existence of bounded, periodic and almost periodic solutions of systems of ordinary differential equations*, Dokl. Akad. Nauk SSSR **123** (1958), 235–238. (Russian)

134. ———, *On some criteria for the existence of periodic solutions for systems of ordinary differential equations*, in Proc. Internat. Sympos. Nonlinear Oscillations, Kiev, vol. 2, Izdat Akad. Nauk Ukrain. SSSR, Kiev, 1963, pp. 202–211. (Russian. English summary)

135. M. A. Krasnosel'skiĭ and V. V. Strygin, *Computation of the rotation of completely continuous vector fields connected with the problem of periodic solutions of differential equations,* Soviet Math. Dokl. 4 (1963), 1351–1354.

136. ———, *Certain tests for the existence of periodic solutions for ordinary differential equations,* Soviet Math. Dokl. 5 (1964), 763–766.

137. M. A. Krasnosel'skiĭ and P. P. Zabreiko, *Geometrical methods in nonlinear analysis,* Nauka, Moscow, 1975. (Russian)

138. L. Kronecker, *Uber Systeme von Funktionen mehrer Variabeln,* Monatsberichte Akad. Wiss. Berlin, 1869, pp. 159–193; 688–698.

139. B. Laloux, *Indice de coïncidence et bifurcations,* in Equations différentielles et fonctionnelles non linéaires, P. Janssens, J. Mawhin and N. Rouche, ed., Hermann, Paris, 1973, pp. 109–121.

140. ———, *Indice de coïncidence et bifurcations,* Thèse de Doctorat, Univ. de Louvain, 1974.

141. ———, *Periodic solutions of some autonomous ordinary differential equations,* Ann. Soc. Sci. Bruxelles Sér. I 91 (1977), 62–68.

142. ———, *Multiplicity and bifurcation of periodic solutions in ordinary differential equations,* Proc. Roy. Soc. Edinburgh Sect. A Math. (to appear).

143. ———, *On the Hopf bifurcation theorem,* in Equadiff 78, Convegno Internaz. Equaz. Diff. Ordin. ed Equaz. Funzionali, R. Conti, G. Sestini and G. Villari, ed., Univ. of Firenze, 1978, Supplemento, pp. 25–34.

144. B. Laloux and J. Mawhin, *Coincidence index and multiplicity,* Trans. Amer. Math. Soc. 217 (1976), 143–162.

145. ———, *Multiplicity, Leray-Schauder formula and bifurcation,* J. Differential Equations 24 (1978), 309–322.

146. C. C. Lan, *Boundary value problems for second and third order differential equations,* J. Differential Equations 18 (1975), 258–274.

147. E. M. Landesman and A. C. Lazer, *Nonlinear perturbations of linear elliptic boundary value problems at resonance,* J. Math. Mech. 19 (1970), 609–623.

148. A. C. Lazer, *Topological degree and symmetric families of periodic solutions of nondissipative second-order systems,* J. Differential Equations 19 (1975), 62–69.

149. ———, *Some resonance problems for elliptic boundary value problems,* in Nonlinear Functional Analysis and Differential Equations, L. Cesari, R. Kannan and J. D. Schuur, ed., Dekker, New York, 1977, pp. 269–289.

150. A. C. Lazer and D. E. Leach, *Bounded perturbations of forced harmonic oscillations at resonance,* Ann. Mat. Pura Appl. 82 (1969), 49–68.

151. M. Lees, *A boundary value problem for nonlinear ordinary differential equations,* J. Math. Mech. 10 (1961), 423–430.

152. J. Leray, *Etude de diverses équations intégrales non linéaires et de quelques problemes que pose l'hydrodynamique,* J. Math. Pures Appl. 12 (1933), 1–82.

153. ———, *Topologie des espaces abstraits de M. Banach,* C. R. Acad. Sci. Paris 200 (1935), 1082–1084.

154. J. Leray, *Les problèmes non linéaires,* Enseignement Math. **35** (1936), 139–151.

155. J. Leray and J. Schauder, *Topologie et équations fonctionnelles,* Ann. Sci. École Norm. Sup. **51** (1934), 45–78.

156. E. A. Lifschits, *Duality principles for the problem of periodic solutions of higher-order differential equations,* Soviet Math. Dokl. **6** (1965), 1442–1445.

157. A. M. Lyapunov, *Problème général de la stabilité du mouvement,* Ann. Fac. Sci. Toulouse **9** (1907), 203–474.

158. ———, *Sur les figures d'équilibre peu différentes des ellipsoïdes d'une masse liquide homogène dotée d'un mouvement de rotation,* Zap. Akad. Nauk St. Petersbourg, 1906, 1–225; 1908, 1–175; 1912, 1–228; 1914, 1–112.

159. R. J. Magnus, *A generalization of multiplicity and the problem of bifurcation,* Proc. London Math. Soc. **32** (1976), 251–278.

160. V. P. Maksimov, *The Noether property of a boundary value problem for a linear functional-differential equation,* Differential Equations **10** (1974), 1771–1774.

161. M. Martelli, *Spectral theory for nonlinear operators and forced second-order nonlinear oscillations,* in Equadiff 78, Convegno Internaz. Equaz. Diff. Ordin. ed Equaz. Funzionali, R. Conti, G. Sestini and G. Villari, ed., Univ. of Firenze, 1978, pp. 501–514.

162. J. Mawhin, *Degré topologique et solutions périodiques des systèmes différentiels non linéaires,* Bull. Soc. Roy. Sci. Liège **38** (1969), 308–398.

163. ———, *Equations intégrales et solutions périodiques des systèmes différentiels non linéaires,* Acad. Roy. Belg. Bull. Cl. Sci. **55** (1969), 934–947.

164. ———, *Existence of periodic solutions for higher order differential systems that are not of class D,* J. Differential Equations **8** (1970), 523–530.

165. ———, *Periodic solutions of nonlinear functional differential equations,* J. Differential Equations **10** (1971), 240–261.

166. ———, *Equations non linéaires dans les espaces de Banach,* Sém. Math. Appl. Méc. Univ. Cath. Louvain, Rapport no. 39, Vander, Louvain, 1971.

167. ———, *Equivalence theorems for nonlinear operator equations and coincidence degree theory for some mappings in locally convex topological vector spaces,* J. Differential Equations **12** (1972), 610–636.

168. ———, *A further invariance property of coincidence degree in convex spaces,* Ann. Soc. Sci. Bruxelles Sér. I **87** (1973), 51–57.

169. ———, *Nonlinear perturbations of Fredholm mappings in normed spaces and applications to differential equations,* Univ. de Brasilia, Trabalho de Matematica no. 61, 1974.

170. ———, *Topology and nonlinear boundary value problems,* in Dynamical Systems: An International Symposium, L. Cesari, J. K. Hale and J. P. LaSalle, ed., vol. I, Academic Press, New York, 1976, pp. 51–82.

171. ———, *Functional analysis and boundary value problems,* in Studies in Ordinary Differential Equations, J. K. Hale, ed., Math. Assoc. America, Washington, D. C., 1977, pp. 128–168.

172. ———, *Landesman-Lazer's type problems for nonlinear equations,* Conf. Sem. Mat. Univ. Bari, no. 147, 1977.

173. J. Mawhin, *Boundary value problems at resonance for vector second order nonlinear ordinary differential equations,* in Proc. Equadiff IV, Praha, 1977, Lecture Notes in Math., Springer-Verlag, Berlin and New York (to appear).

174. J. Mawhin and K. Schmitt, *Rothe and Altman type coincidence theorems and applications to differential equations,* Nonlinear Analysis, Theory, Methods and Appl. 1 (1977), 151–160.

175. J. Mawhin and W. Walter, *Periodic solutions of ordinary differential equations with one-sided growth restrictions,* Proc. Roy. Soc. Edinburgh Sect. A Math. 82A (1978) (in press).

176. J. Mawhin and M. Willem, *Periodic solutions of nonlinear differential equations in Hilbert spaces,* in Equadiff 78, Convegno Internaz. Equaz. Diff. Ordin. ed Equaz. Funzionali, R. Conti, G. Sestini and G. Villari, ed., Univ. of Firenze, 1978, pp. 323–332.

177. J. W. Milnor, *Topology from the differentiable viewpoint,* Univ. Press of Virginia, Charlottesville, 1965.

178. M. Mininni, *Coincidence degree and solvability of some nonlinear functional equations in normed spaces: a spectral approach,* Nonlinear Analysis, Theory, Methods and Appl. 1 (1977), 105–122.

179. ———, *A spectral approach to local solvability of some nonlinear equations in normed spaces,* Nonlinear Analysis, Theory, Methods and Appl. 2 (1978), 597–607.

180. C. Mininni, *About the solvability of some nonlinear equations with α-contractivity conditions,* Boll. Un. Mat. Ital. (to appear).

181. C. Miranda, *Partial differential equations of elliptic type,* Springer-Verlag, Berlin, 1970.

182. J. Moser, *Periodic solutions near an equilibrium and a theorem by Alan Weinstein,* Comm. Pure Appl. Math. 29 (1976), 727–747.

183. E. M. Muhamadiev and B. N. Sadovskii, *Estimation of the spectral radius of a certain operator that is connected with equations of neutral type,* Math. Notes 13 (1973), 39–45.

184. J. Myjak, *A boundary value problem for nonlinear differential equations with a retarded argument,* Ann. Polon. Math. 27 (1973), 133–141.

185. ———, *Existence and uniqueness of solutions of boundary value problems for functional differential equations,* Zeszyty Nauk Univ. Jagiellon. Prace Mat. 16 (1974), 137–146.

186. ———, *Boundary value problems for functional differential equations,* Ann. Polon. Math. 32 (1976), 23–33.

187. J. McBain, *Local and global bifurcation from normal eigenvalues,* Pacific J. Math. 63 (1976), 445–466.

188. M. Nagumo, *A theory of degree based on infinitesimal analysis,* Amer. J. Math. 73 (1951), 485–496.

189. ———, *Degree of mapping in convex linear topological spaces,* Amer. J. Math. 73 (1951), 497–511.

190. M. Z. Nashed, *Generalized inverses and applications,* Academic Press, New York, 1976.

191. L. Nirenberg, *Functional analysis,* New York Univ. Lecture Notes, New York, 1960–1961.

192. ——, *Topics in nonlinear functional analysis,* New York Univ. Lecture Notes, New York, 1973–74.

193. V. R. Nosov, *Sur l'alternative de Fredholm pour les systèmes linéaires généraux à coéfficients périodiques et à argument déplacé,* C. R. Acad. Sci. Paris **270** (1970), 1097–1100.

194. ——, *Periodic solutions of systems of general linear equations with a deviating argument,* Differential Equations **7** (1971), 492–500.

195. ——, *The existence of periodic solutions for linear systems of general form with distributed deviation of the argument,* Differential Equations **7** (1971), 2168–2175.

196. R. D. Nussbaum, *Asymptotic fixed point theorems for local condensing maps,* Math. Ann. **191** (1971), 181–195.

197. ——, *Some asymptotic fixed point theorems,* Trans. Amer. Math. Soc. **171** (1972), 349–375.

198. ——, *Degree theory for local condensing maps,* J. Math. Anal. Appl. **37** (1972), 741–766.

199. ——, *Periodic solutions of some nonlinear autonomous functional differential equations.* I, Ann. Mat. Pura Appl. **10** (1974), 263–306; II, J. Differential Equations **14** (1973), 360–394.

200. ——, *A global bifurcation theorem with applications to functional differential equations,* J. Functional Analysis **19** (1975), 319–338.

201. K. J. Palmer, *On the existence of periodic solutions of a nonlinear vector differential equation,* J. Math. Anal. Appl. **64** (1978), 328–333.

202. H. O. Peitgen, *Some applications of the fixed point index in asymptotic fixed point theory,* in Fixed Point Theory and its Applications, Academic Press, New York, 1976, pp. 137–148.

203. W. V. Petryshyn, *Existence theorems for semilinear abstract and differential equations with noninvertible linear parts and noncompact perturbations,* in Proc. Sympos. Nonlinear Equations in Abstract Spaces, Academic Press, New York, 1978.

204. E. Picard, *Sur le nombre de racines communes à plusieurs équations simultanées,* J. Math. Pures Appl. **8** (1892), 5–24.

205. ——, *Sur l'application des méthodes d'approximations successive à l'étude de certaines équations différentielles ordinaires,* J. Math. Pures Appl. **9** (1893), 217–271.

206. V. A. Pliss, *Families of periodic solutions of dissipationless systems of second order differential equations,* Differential Equations **1** (1965), 1122–1138.

207. E. Podolak, *On asymptotic nonlinearities,* J. Differential Equations **26** (1977), 69–79.

208. H. Poincaré, *Sur les courbes définies par une équation différentielle,* J. Math. Pures Appl. **7** (1881), 375–422; **8** (1882), 251–296; **1** (1885), 167–244; **2** (1886), 151–217.

209. ——, *Sur l'équilibre d'une masse fluide,* Acta Math. **7** (1885), 259–302.

210. H. Poincaré, *Les méthodes nouvelles de la mécanique céleste,* Gauthier-Villars, Paris, 3 volumes, 1892–1899.

211. P. H. Rabinowitz, *Some global results for nonlinear eigenvalue problems,* J. Functional Analysis **7** (1971), 487–513.

212. ——, *Global theorems for nonlinear eigenvalue problems and applications,* in Contributions to Nonlinear Functional Analysis, E. H. Zarantonello, ed., Academic Press, New York, 1971, pp. 11–36.

213. ——, *Some aspects of nonlinear eigenvalue problems,* Rocky Mountain J. Math. **3** (1973), 161–202.

214. ——, *A survey of bifurcation theory,* in Dynamical Systems: An International Symposium, vol. 1, L. Cesari, J. K. Hale, J. P. LaSalle, ed., Academic Press, New York, 1976, pp. 83–96.

215. ——, *Théorie du degré topologique et applications à des problémes aux limites non linéaires,* Université de Paris VI, Laboratoire d'Analyse Numérique, Paris, 1975.

216. W. T. Reid, *Ordinary differential equations,* Wiley, New York, 1971.

217. R. Reissig, *Ein funktionalanalytischer Existenzbeweis für periodische Lösungen,* Monatsber. Deutsch. Akad. Wiss. Berlin **6** (1964), 407–413.

218. ——, *An extension of Ezeilo's result,* Ann. Mat. Pura Appl. **112** (1972), 199–210.

219. ——, *On a general type of second order forced nonlinear oscillations,* in Dynamical Systems: An International Symposium, L. Cesari, J. K. Hale, J. P. LaSalle, ed., vol. 2, Academic Press, New York, 1976, pp. 223–226.

220. ——, *On a certain second order differential equation,* Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. **60** (1976), 395–399.

221. ——, *Periodic solutions of certain nonlinear systems,* in Equadiff 78, Convegno Internaz. Equaz. Diff. Ordin. ed Equaz. Funzionali, R. Conti, G. Sestini and G. Villari, ed., Univ. of Firenze, Firenze, 1978, pp. 1–7.

222. G. E. H. Reuter, *Note on the existence of periodic solutions of certain differential equations,* unpublished.

223. E. Rothe, *Zur Theorie der topologischer Ordnung und der Vektorfelder in Banachsen Räumen,* Compositio Math. **5** (1937), 177–197.

224. N. Rouche and J. Mawhin, *Equations différentielles ordinaires,* 2 volumes, Masson, Paris, 1973.

225. B. N. Sadovskii, *Limit-compact and condensing operators,* Russian Math. Surveys **27** (1972), 85–156.

226. H. Schaefer, *Uber die Methode der a priori-Schranken,* Math. Ann. **129** (1955), 415–416.

227. J. Schauder, *Der Fixpunktsatz in Funktionalräumen,* Studia Math. **2** (1930), 171–180.

228. M. Schechter, *Principles of functional analysis,* Academic Press, New York, 1973.

229. E. Schmidt, *Zur Theorie des linearen und nichtlinearen Integralgleichungen. 3* Teil. Uver die Auflösung der nichtlinearen Integralgleichungen und die Verzweigung ihrer Lösungen, Math. Ann. **65** (1908), 370–399.

230. K. Schmitt, *Applications of variational equations to ordinary and partial differential equations–Multiple solutions of boundary value problems*, J. Differential Equations **17** (1975), 154–186.

231. ———, *Fixed point and coincidence theorems with applications to nonlinear differential and integral equations*, Univ. Cath. de Louvain, Rapport Sémin. Math. Appl. Mécanique, no. 97, Vander, Louvain, 1976.

232. ———, *Boundary value problems for quasilinear second order elliptic equations*, Nonlinear Analysis, Theory, Methods and Appl. **2** (1978), 263–309.

233. ———, *Periodic solutions of a forced nonlinear oscillator involving a onesided restoring force*, Arch. Math. (Basel) **31** (1978), 70–73.

234. K. R. Schneider, *Über eine Integralgleichungsmethode in der Theorie autonomer Schwingungen und ihre Anwendung auf Bifurkationsprobleme*, Math. Nachr. **69** (1975), 191–204.

235. ———, *Investigations of integral equations in the theory of nonlinear oscillations*, in VII. Internat. Konf. Nichtlineare Schwingungen, Band I.2, Abhandl. Akad. Wiss. Deutsche Demokr. Republ., 1977, pp. 221–236.

236. K. R. Schneider and B. Wegner, *Asymptotic behavior of the period of a closed trajectory contracting to a singular point*, in VII. Internat. Konf. Nichtlineare Schwingungen, Band I.2, Abhandl. Akad. Wiss. Deutsche Demokr. Republ., 1977, pp. 215–219.

237. J. Schroder, *Pointwise norm bounds for systems of ordinary differential equations*, Math. Inst. Univ. Köln, Angew. Math. Report 77–14, 1977.

238. ———, *Two-sided bounds and norm bounds for systems of nonlinear ordinary differential equations*, Math. Inst. Univ. Köln, Angew. Math. Report 77–16, 1977.

239. J. D. Schuur, *Boundary value problems for $x'' = f(t, x, x')$ and nonlinear functional analysis*, in Nonlinear Functional Analysis and Differential Equations, L. Cesari, R. Kannan and J. D. Schuur, ed., Dekker, New York, 1977, pp. 319–337.

240. ———, *Perturbation at resonance for a fourth order ordinary differential equation*, J. Math. Anal. Appl. **65** (1978), 20–25.

241. J. T. Schwartz, *Nonlinear functional analysis*, Gordon and Breach, New York, 1969.

242. H. W. Siegberg, *Abbildungsgrade in Analysis und Topologie*, Diplomarbeit, Bonn Universität, Bonn, 1977.

243. C. Siegel and J. Moser, *Lectures on celestial mechanics*, Springer-Verlag, New York, 1971.

244. S. Smale, *An infinite dimensional version of Sard's theorem*, Amer. J. Math. **87** (1965), 961–966.

245. E. H. Spanier, *Algebraic topology*, McGraw-Hill, New York, 1966.

246. F. Stoppelli, *Su un'equazione differenziale della meccanica dei fili*, Rend. Accad. Sci. Fis. Mat. Napoli **19** (1952), 109–114.

247. V. V. Strygin, *The relationship theorem for the periodic problem of an autonomous system of ordinary differential equations*, in Proc. Fifth Internat. Conf. Nonlinear Oscillations, vol. 1, Izdat Akad. Nauk Ukrain. SSR., Kiev, 1970, pp. 535–539. (Russian)

248. C. A. Stuart, *Three fundamental theorems on bifurcation*, Ecole Polytech. de Lausanne, Dép. de Math., 1977.

249. J. Tippett, *An existence-uniqueness theorem for two-point boundary value problems*, SIAM J. Math. Anal. **5** (1974), 153–157.

250. M. M. Vainberg and P. G. Aizengendler, *The theory and methods of investigations of branch points of solutions*, in Progress in Math., R. V. Gamkrelidze, ed., vol. 2, Plenum Press, New York, 1968, pp. 1–72.

251. M. M. Vainberg and V. A. Trenogin, *The method of Lyapunov and Schmidt in the theory of nonlinear differential equations and their further development*, Russian Math. Surveys **17** (1962), 1–60.

252. ——, *Theory of branching of solutions of non-linear equations*, Noordhoff, Leyden, 1974.

253. G. Villari, *Contributi allo studio dell'esistenza di soluzioni periodiche per i sistemi di equazioni differenziali ordinarie*, Ann. Mat. Pura Appl. **69** (1965), 171–190.

254. ——, *Soluzioni periodiche di una classe di equazioni differenziali del terz'ordine quasi lineari*, Ann. Mat. Pura Appl. **73** (1966), 103–110.

255. P. Waltman and J. S. W. Wong, *Two point boundary value problems for nonlinear functional differential equations*, Trans. Amer. Math. Soc. **164** (1972), 39–54.

256. J. Weinberg, *A topological degree of locally A-proper maps*, J. Math. Anal. Appl. **65** (1978), 66–79.

257. D. Weistreich, *Bifurcation at eigenvalues of odd multiplicity*, Proc. Amer. Math. Soc. **41** (1973), 609–614.

258. ——, *Periodic solutions of second-order non-Lagrangian systems*, Israel J. Math. **16** (1973), 279–286.

259. ——, *Periodic solutions of second order Lagrangian systems*, Duke Math. J. **41** (1974), 405–411.

260. ——, *A variational theorem for nonlinear differential equations*, Indiana Univ. Math. J. **25** (1976), 1149–1157.

261. ——, *Global analysis and periodic solutions of second order systems of nonlinear differential equations*, Pacific J. Math. **67** (1976), 537–554.

262. J. Werner, *Periodische Lösungen bei Systemen von Differentialgleichungen zweiter Ordnung*, in Internat. Sch. Numer. Math., Band 19, Birkhauser, Basel, 1974, pp. 213–218.

263. D. Wexler, *Solutions périodiques des systèmes linéaires à argument retardé*, J. Differential Equations **3** (1967), 236–247.

264. E. Winston, *An existence theorem for a periodic solution of perturbed periodic systems*, in Seminar on Differential Equations and Dynamical Systems, II, Lecture Notes in Math., vol. 144, Springer-Verlag, Berlin and New York, 1970, pp. 262–266.

265. ——, *The existence of periodic solutions of perturbed periodic systems*, SIAM J. Appl. Math. **21** (1971), 95–103.

266. E. Zeidler, *Existenz, Eindeutigkeit, Eigenschaften und Anwendungen des Abbildungsgrades in R^n*, in Theory of Nonlinear Operators, Proc. of a Summer School, Akademie-Verlag, Berlin, 1974, pp. 259–311.

267. ——, *Vorlesungen über Nichtlineare Funktionalanalysis. I. Fixpunktsätze*, Teubner Text in Mathematik, Teubner, Leipzig, 1976.