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Lectures on Three-Manifold Topology

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PREFACE

This manuscript is a detailed presentation of the ten lectures given by the author at the Conference Board of Mathematical Sciences Regional Conference on Three-Manifold Topology, held October 8-12, 1977, at Virginia Polytechnic Institute and State University. The purpose of the conference was to present the current state of affairs in three-manifold topology and to integrate the classical results with the many recent advances and new directions.

The ten principal lectures presented here deal with the study of three-manifolds via incompressible surfaces. At the conference, these lectures were supplemented by eight special lectures: two lectures each by Professor Joan Birman of Columbia University (Heegaard Theory), Professor Sylvain Cappell of the Courant Institute (branched coverings and applications to 4-dimensional topology), Professor Robion Kirby of The University of California/Berkeley (three-dimensional knot theory) and Professor William Thurston of Princeton University (three-dimensional hyperbolic geometry).

I wish to thank the host university, the members of the Department of Mathematics at V.P.I. & S.U., and the Conference Director, Professor Charles Feustel. A special thanks to the Conference Coordinator, Professor Ezra 'Bud' Brown.

I believe that the conference was a success and that much of the success was due to the special lectures given by the previously named participants. I wish to acknowledge my thanks to Professors Birman, Cappell, Kirby and Thurston.

In the preparation of my lectures I had many helpful discussions with my close colleagues, Benny Evans, John Hempel and Peter Shalen. Their help has been extended to the completion of the writing of these lecture notes. In the preparation of the manuscript, I have been assisted by John Rice, the staff at The Institute for Advanced Study, especially Elizabeth Gorman Moyer and Gail Sydow, and at Rice University by Anita Poley.

I dedicate this manuscript to my wife, Linda, who has been understanding and constantly supportive over the long period of time that I have spent in its preparation.

William Jaco

Houston, Texas

July, 1979

INTRODUCTION

This manuscript is intended to present the development of three-manifold topology evolving from the study of incompressible surfaces embedded in three-manifolds. This, of course, is quite a restriction to come under the broad title of the manuscript. But even here, the reader will find many important aspects in the theory of incompressible surfaces missing. I am not trying to make the manuscript all inclusive (I do not believe that a possible task) and I have not tried to make the bibliography complete. The manuscript is exactly what I would do in ten lectures with the above intention.

The reader will find the subject, through the first six chapters, overlappling with the book by John Hempel, [He_1]. I have very high regard for Hempel's book and debated a bit about presuming its contents. However, I decided that this manuscript would better serve if the development began more at the foundations. And in the end, I believe that the reader will find the overlap mostly in spirit and terminology. I have developed the material from my point of view and I give a number of new proofs to the classical results. If I needed material that appears in Hempel's book, and if I felt that the development there was consistent with my development, then I refer the reader to the appropriate result. While this manuscript certainly can be considered independent of Hempel's book, it can also be considered as a sequel to it.

In Chapter I, I wanted to give a unified proof of the Loop Theorem - Dehn's Lemma and the Sphere Theorem, using equivarient surgery. I also had another motive for this approach; namely, to prove that the universal covering space of an orientable, irreducible 3-manifold is itself irreducible. I was only able to carry through equivarient surgery in the limited case of two-sheeted coverings (involutions); hence, I give an equivarient surgery proof of the Loop-Theorem and Dehn's Lemma; but I give no new information on the Sphere Theorem. Since I presented these lectures, W. Meeks and S. T. Yau have given unified proofs of the Loop Theorem - Dehn's Lemma and the Sphere Theorem, using minimal surfaces to accomplish equivarient surgery. Their methods also show that the universal covering space of an orientable, irreducible 3-manifold is irreducible.

In Chapter II the main result is the Prime Decomposition Theorem (II.4) for compact, orientable 3-manifolds. Here, the classical proof of existence is due to H. Kneser [Kn_1] and is a very intriguing proof. However, a more natural approach is an argument based on reasoning by induction. I present such an argument using a theorem of W. Haken [Ha_1], which states that a closed, orientable 3-manifold admitting a connected sum decomposition, admits a connected sum decomposition by closed, orientable 3-manifolds having strictly smaller Heegaard genus (II.7). Even here I give a new proof of Haken's Theorem, which is surprisingly easy and, moreover, is really a 2-dimensional argument.

In Chapter III, I give the definition of an incompressible surface and of a Haken-manifold. I give some sufficient (and some necessary) conditions for the existence of an incompressible surface in a 3-manifold. I have included a number of examples, many of which I use later in the manuscript. Here, however, the main result is the so-called Haken-Finiteness Theorem (III.20). It gives a finiteness condition for collections of pairwise disjoint, incompressible surfaces embedded in a Haken-manifold. It is indespensible to the approach of this author and I have presented it in both detail and generality. I also present a convenient generalization due to Peter Shalen and myself.

I believe the reader will find that Chapter IV is a fun chapter. It is a very important chapter; since the existence of a hierarchy for a Haken-manifold provides an inductive method of proof, which has been a major tool employed in the study of this important class of 3-manifolds. But, in this chapter I introduce the notion of a partial hierarchy and give some fun examples of infinite partial hierarchies for compact 3-manifolds. I also define the (closed) Haken number of a compact 3-manifold and the length of a Haken-manifold; and I discuss different inductive methods of proof (advantages and perils). I end Chapter IV with Theorem IV.19, where I prove that any Haken-manifold has a hierarchy of length no more than four — a result that I have never been able to use.

Chapter V is an abbreviated version of what might have been a revision of my Princeton Lecture Notes on the structure of three-manifold groups. I first indicate the restrictive nature of three-manifold groups by classifying the abelian three-manifold groups. I present the Scott-Shalen Theorem (V.16) that any finitely generated three-manifold group is finitely presented. This is done using the idea of indecomposably covering a group, which I like very much. However, a large part of Chapter V is devoted to open questions about three-manifold groups and properties for three-manifold groups; e.g., the finitely generated intersection property for groups, which is important in later chapters (particularly, Chapter VII).

Chapter VI is in some sense the beginning of the new material. Here I present new results about Seifert fibered manifolds. Namely, I prove that if a finite sheeted covering space of a Haken manifold M is a Seifert fibered manifold, then M itself is a Seifert fibered manifold (VI.29) and I develop the topological study of Seifert fibered manifolds needed in the later chapters to present the recent work of Shalen and myself, Waldhausen and Johannson. Also, I present a proof of the Gordon-Heil prediction that a Haken-manifold, having an infinite cyclic, normal subgroup of its fundamental group, is a Seifert fibered manifold (VI.24) and a description of compact, incompressible surfaces in Seifert fibered manifolds (VI.34). I hope that this chapter on Seifert fibered manifolds will serve as an introduction to this important class of 3-manifolds for the beginners in the subject and provide some enjoyable reading for the more advanced.

I begin Chapter VII by giving general conditions that are sufficient for a noncompact 3-manifold to admit a manifold compactification. The basic result here (VII.1) is after T. Tucker's work [Tu_1]. I give a new proof of J. Simon's Theorem [Si_1] that the covering space of a Haken-manifold corresponding to the conjugacy class of a subgroup of the fundamental group, which has the finitely generated intersection property, admits a manifold-compactification to a Haken-manifold (VII.4). This result is then applied to covering spaces corresponding to finitely generated peripheral subgoups (finitely generated, peripheral subgroups have the finitely generated intersection property (V.20), [J-M] and [$J\text{-}S_2$]) and to covering spaces corresponding to the fundamental groups of well-embedded submanifolds. This latter material is based on work of B. Evans and myself and allows certain isotopes that are used in all the later chapters. While this work on compactifications is itself very important to me, its use here provides the foundation for the existence and uniqueness of the characteristic pair factor of a Haken-manifold pair. This is a new presentation of the main results of the paper [J-S_2]. I use covering space arguments rather than the formal homotopy language used there. I believe that the reader should find the material here much more intuitive. I have not, however, covered many of the surprising results obtained in that manuscript. Time and space would not allow this. I am disappointed in not doing so; but if the reader has interest after completing Chapter VII, I believe revisiting [J-S_2] may be a more pleasant experience.

In Chapter VIII, I present a major part of the joint work of Shalen and myself [J-S_1]. My approach here follows very much the lines of our original approach. While this approach is similar to the presentation in [J-S_1], I do not go into the generalities of that manuscript (thereby, I considerably reduce the notation and length of the presentation) and I have set up a different foundation with the material of Chapter VII. I give proofs of the Essential Homotopy Theorem (VIII.4), the Homotopy Annulus Theorem (VIII.10) and the Homotopy Torus Theorem (VIII.11) and the Annulus-Torus Theorems (VIII.13 and VIII.14).

I believe that the presentation of Chapter IX will introduce the reader to an understanding of the characteristic Seifert pair of a Haken-manifold with a minimum amount of work. In fact, this chapter is really very short. By introducing the idea of a perfectly-embedded Seifert pair in a 3-manifold, I am able to show (under the partial ordering that one perfectly-embedded Seifert pair (Σ',Φ') is less than or equal to another perfectly-embedded Seifert pair (Σ,Φ) if there is an ambient isotopy taking Σ' into $\text{Int}\ \Sigma$ and Φ' into $\text{Int}\ \Phi$) that a Haken-manifold with incompressible boundary admits a unique (up to ambient isotopy) maximal, perfectly-embedded Seifert pair. This unique maximal, perfectly-embedded Seifert pair is the characteristic Seifert pair. The bulk of Chapter IX is in giving a detailed proof of Theorem IX.17, which allows a characterization of the characteristic Seifert pair via homotopy classifications of maps. I conclude Chapter IX with examples of the characteristic Seifert pair of some familiar 3-manifolds and some examples for caution.

I think it is fair to say that I worked the hardest for the results of Chapter X. I guess what is good about this is that I am pleased with the outcome. For here, I present the generalized version of Waldhausen's Theorem on deforming homotopy equivalences (X.7); hence, I have answered conjecture 13.10 of [He_1] affirmatively and given a complete proof of this version of the theorem, which is due to T. Tucker (X.9). Moreover, I have been able to obtain a new proof of the beautiful theorem due originally to K. Johannson [Jo_2] on the deformations of homotopy equivalences between Haken-manifolds with incompressible boundary (X.15 and X.21). This proof is inspired by observations of A. Swarup (VII.22 and VII.24). I also give many examples of "exotic" homotopy equivalences. Most of these examples are well-known.

I have used standard terminology and notation except for the new terms that are introduced; of course, here I give the required definitions and describe the new notation. The reader will find that I have included a lot of detail. However, if there were items that I felt might improve the presentation and I did not want to complete the detail, I set such items off as Exercises. This is in contrast to items labeled as Question; in the case of a question, I simply do not know the answer.

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