Conference Board of the Mathematical Sciences

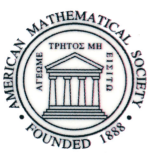
CBMS

Regional Conference Series in Mathematics

Number 48

Brown-Peterson Homology: An Introduction and Sampler

W. Stephen Wilson



American Mathematical Society
with support from the
National Science Foundation

BROWN-PETERSON HOMOLOGY: AN INTRODUCTION AND SAMPLER

This page intentionally left blank

http://dx.doi.org/10.1090/cbms/048

Conference Board of the Mathematical Sciences
REGIONAL CONFERENCE SERIES IN MATHEMATICS

supported by the
National Science Foundation

Number 48

BROWN-PETERSON HOMOLOGY: AN INTRODUCTION AND SAMPLER

by
W. STEPHEN WILSON

**Published for the
Conference Board of the Mathematical Sciences
by the
American Mathematical Society
Providence, Rhode Island**

Expository Lectures
from the CBMS Regional Conference
held at the State University of New York, Center at Albany
June 23–27, 1980

2000 *Mathematics Subject Classification.* Primary 55–XX.

Library of Congress Cataloging-in-Publication Data

Wilson, W. Stephen, 1946–
Brown-Peterson homology.
(Regional conference series in mathematics; no. 48)
"Expository lectures from the CBMS regional conference held at the State University of New York, Center at Albany, June 23–27, 1980"—T.p. verso.
Includes bibliographical references.
1. Homology theory. 2. Homotopy theory. I. Conference Board of the Mathematical Sciences. II. Title. III. Series.

QA1.R33 no. 48 [QA612.3] 510s [514'.23] 81-20619
ISBN 0-8218-1699-3 AACR2
ISSN 0160-7642

---

10 9 8 7 6 5 4 3 2 10 09 08 07 06 05

Contents

This page intentionally left blank

This page intentionally left blank

References

[A_1] J. F. Adams, *Lectures on generalized cohomology,* Lecture Notes in Math., vol. 99, Springer-Verlag, Berlin, 1969, pp. 1–138.

[A_2] ———, *Quillen's work on formal groups and complex cobordism,* Stable Homotopy and Generalized Homology, Univ. of Chicago Press, Chicago, Ill., 1974, pp. 29–120.

[A_3] ———, *On the groups $J(X)$. IV,* Topology **5** (1966), 21–71.

[A_4] ———, *Localization and completion with an addendum on the use of Brown-Peterson homology in stable homotopy,* Univ. of Chicago Lecture Notes in Mathematics, 1975.

[Ar_1] S. Araki, *Typical formal groups in complex cobordism and K-theory,* Lectures in Math., Dept. of Math., Kyoto Univ., Tokyo, Kinokuniya Book Store, 1973.

[Ar_2] ———, *Multiplicative operations in BP cohomology,* Osaka J. Math. **12** (1975), 343–356.

[Ba] N. A. Baas, *On bordism theory of manifolds with singularities,* Math. Scand. **33** (1973), 279–302.

[BCM] M. Bendersky, E. B. Curtis and H. R. Miller, *The unstable Adams spectral sequence for generalized homology,* Topology **17** (1978), 229–248.

[Br] E. H. Brown, *Cohomology theories,* Ann. of Math. (2) **75** (1962), 467–484.

[BP] E. H. Brown and F. P. Peterson, *A spectrum whose Z_p-cohomology is the algebra of reduced p-th powers,* Topology **5** (1966), 149–154.

[Ch] K. Chan, *Applications of the bar and cobar spectral sequences to the Brown-Peterson spectrum,* Thesis, The Johns Hopkins University, 1980.

[C] P. E. Conner, *Differentiable periodic maps,* 2nd ed., Graduate Texts in Math., no. 738, Springer-Verlag, Berlin, 1978.

[CF_1] P. E. Conner and E. E. Floyd, *Differential periodic maps,* Springer-Verlag, Berlin, 1964.

[CF_2] ———, *The relation of cobordism to K-theories,* Lecture Notes in Math., vol. 28, Springer-Verlag, Berlin, 1966.

[CF_3] ———, *Torsion in SU-bordism,* Mem. Amer. Math. Soc., no. 60, Amer. Math. Soc., Providence, R. I., 1966.

[CS] P. E. Conner and L. Smith, *On the complex bordism of finite complexes,* Inst. Hautes Études Sci. Publ. Math. **37** (1969), 117–221.

[tD] T. tomDieck, *Actions of finite abelian p-groups without stationary points,* Topology **9** (1970), 359–366.

[F] E. E. Floyd, *Actions of $(Z_p)^k$ without stationary points*, Topology **10** (1971), 327–336.

[G] V. Giambalvo, *Some tables for formal groups and BP*, Geometric Applications of Homotopy Theory. II, Lecture Notes in Math., vol. 658, Springer-Verlag, Berlin, 1978.

[H_1] M. Hazewinkel, *A universal formal group and complex cobordism*, Bull. Amer. Math. Soc. **81** (1975), 930–933.

[H_2] ———, *Constructing formal groups. III. Applications to complex cobordism and Brown-Peterson cohomology*, J. Pure Appl. Algebra **10** (1977/78), 1–18.

[JMWZ] D. C. Johnson, H. R. Miller, W. S. Wilson and R. S. Zahler, *Boundary homomorphisms in the generalized Adams spectral sequence and the non-triviality of infinitely many γ_t in stable homotopy*, Reunion Sobre Teoria de Homotopia (Donald Davis, ed.), Notas de Matemáticas y Simposia, no. 1, Sociedad Matemáticas Mexicana, Mexico City, 1975, pp. 47–59.

[JW_1] D. C. Johnson and W. S. Wilson, *Projective dimension and Brown-Peterson homology*, Topology **12** (1973), 327–353.

[JW_2] ———, *BP operations and Morava's extraordinary K-theories*, Math. Z. **144** (1975), 55–75.

[JY] D. C. Johnson and Z. Yosimura, *Torsion in Brown-Peterson homology and Hurewicz homomorphisms*, Osaka J. Math. **17** (1980), 117–136.

[K] I. Kozma, *Witt vectors and complex cobordism*, Topology **13** (1974), 389–394.

[L_1] P. S. Landweber, *Cobordism operations and Hopf algebras*, Trans. Amer. Math. Soc. **129** (1967), 94–110.

[L_2] ———, *Annihilator ideals and primitive elements in complex bordism*, Illinois J. Math. **17** (1973), 273–284.

[L_3] ———, *Associated prime ideals and Hopf algebra*, J. Pure Appl. Algebra **3** (1973), 43–58.

[L_4] ———, *Homological properties of comodules over MU_*MU and BP_*BP*, Amer. J. Math. **98** (1976), 591–610.

[L_5] ———, *New applications of commutative algebra to Brown-Peterson homology*, Lecture Notes in Math., vol. 741, Springer-Verlag, Berlin, 1979, pp. 449–460.

[Li] A. Liulevicius, *On the algebra $BP_*(BP)$*, Lecture Notes in Math., vol. 249, Springer-Verlag, Berlin, 1971, pp. 47–53.

[ML] S. MacLane, *Categories for the working mathematician*, Springer, Berlin, 1971.

[Ma] J. Martin, *An algorithm which generates basis elements for the homology of the Brown-Peterson spectrum*, Thesis, The Johns Hopkins University, 1981.

[Mg] R. J. Milgram, *The bar construction and abelian H-spaces*, Illinois J. Math. **11** (1967), 242–250.

[M] H. R. Miller, *Some algebraic aspects of the Adams-Novikov spectral sequence*, Thesis, Princeton University, 1974.

[MR] H. R. Miller and D. C. Ravenel, *Morava stabilizer algebras and the localization of Novikov's E_2-term*, Duke Math. J. **44** (1977), 433–447.

[MRW] H. R. Miller, D. C. Ravenel and W. S. Wilson, *Periodic phenomena in the Adams-Novikov spectral sequence*, Ann. of Math. (2) **106** (1977), 469–516.

[MW] H. R. Miller and W. S. Wilson, *On Novikov's* Ext^1 *modulo an invariant prime ideal*, Topology **15** (1976), 131–141.

[Mi_1] J. W. Milnor, *On the cobordism ring* Ω^* *and a complex analogue. I*, Amer. J. Math. **82** (1960), 505–521.

[Mi_2] ———, *The Steenrod algebra and its dual*, Ann. of Math. (2) **67** (1958), 150–171.

[Mo_1] J. Morava, *Structure theorems for cobordism comodules*, preprint.

[Mo_2] ———, *Extensions of cobordism comodules*, preprint.

[Mo_3] ———, *A product for odd-primary bordism of manifolds with singularities*, Topology **18** (1979), 177–186.

[N] S. P. Novikov, *The methods of algebraic topology from the view point of cobordism theories*, Math. USSR-Izv. **1** (1967), 827–913 = Izv. Akad. Nauk SSSR Ser. Mat. **31** (1967), 855–951.

[OT] S. Oka and H. Toda, *Nontriviality of an element in the stable homotopy groups of spheres*, Hiroshima Math. J. **5** (1975), 115–125.

[Q_1] D. Quillen, *On the formal group laws of unoriented and complex cobordism theory*, Bull. Amer. Math. Soc. **75** (1969), 1293–1298.

[Q_2] ———, *Elementary proofs of some results of cobordism theory using Steenrod operations*, Adv. in Math. **7** (1971), 29–56.

[R_1] D. C. Ravenel, *The structure of* BP_*BP *modulo an invariant prime ideal*, Topology **15** (1976), 149–153.

[R_2] ———, *The structure of Morava stabilizer algebras*, Invent. Math. **37** (1976), 109–120.

[R_3] ———, *Localization with respect to certain periodic homology theories*, preprint.

[RW_1] D. C. Ravenel and W. S. Wilson, *The Hopf ring for complex cobordism*, J. Pure Appl. Algebra **9** (1977), 241–280.

[RW_2] ———, *The Morava K-theories of Eilenberg-MacLane spaces and the Conner-Floyd conjecture*, Amer. J. Math. **102** (1980), 691–748.

[RS] M. Rothenberg and N. Steenrod, *The cohomology of classifying spaces of H-spaces*, Bull. Amer. Math. Soc. **71** (1965), 872–875; mimeographed notes, Princeton Univ.

[SY] N. Shimada and N. Yagita, *Multiplications in complex bordism theory with singularities*, Publ. Res. Inst. Math. Sci. **12** (1976), 259–293.

[Si] K. Sinkinson, *The cohomology of certain spectra associated with the Brown-Peterson spectrum*, Duke Math. J. **43** (1976), 605–622.

[Sm_1] L. Smith, *On the complex bordism of finite complexes*, Proc. Advanced Study Institute on Algebraic Topology, vol. III, Various Publication Series No. 13, Matematisk Institute, Aarhus Universitet, 1970, pp. 513–566.

[Sm_2] ———, *On realizing complex bordism modules,* Amer. J. Math. **92** (1970), 793–856.

[Sn] V. P. Snaith, *Algebraic cobordism and K-theory,* Mem. Amer. Math. Soc. No. 221 (1979).

[St] R. Stong, *Notes on cobordism theory,* Math. Notes, Princeton Univ. Press, Princeton, N. J., 1968.

[Su] D. Sullivan, *Singularities in spaces,* Proc. Liverpool Singularities Sympos. II, Lecture Notes in Math., vol 209, Springer-Verlag, Berlin, 1971, pp. 196–207.

[Sw] R. M. Switzer, *Algebraic topology–homotopy and homology,* Springer, Berlin, 1975.

[T] R. Thom, *Quelques proprietes globales des variétés differentiables,* Comment. Math. Helv. **28** (1954), 17–86.

[TZ_1] E. Thomas and R. S. Zahler, *Nontriviality of the stable homotopy element* γ_1, J. Pure Appl. Algebra **4** (1974), 189–203.

[TZ_2] ———, *Generalized higher order cohomology operations and stable homotopy groups of spheres,* Adv. in Math. **20** (1976), 289–328.

[TW] R. W. Thomason and W. S. Wilson, *Hopf rings in the bar spectral sequence,* Quart. J. Math. **31** (1980), 507–511.

[To_1] H. Toda, *On realizing exterior parts of the Steenrod algebra,* Topology **10** (1971), 53–65.

[To_2] ———, *p-primary components of homotopy groups.* IV, Mem. Coll. Sci. Kyoto Univ. Ser. A **32** (1959), 297–332.

[V] R. Vogt, *Boardman's stable homotopy category,* Lecture Notes Series 21, Matematisk Institut, Aarhus Universitet, 1970.

[Wh] G. W. Whitehead, *Generalized homology theories,* Trans. Amer. Math. Soc. **102** (1962), 227–283.

[W_1] W. S. Wilson, *The Ω-spectrum for Brown-Peterson cohomology.* I, Comment. Math. Helv. **48** (1973), 45–55.

[W_2] ———, *The Ω-spectrum for Brown-Peterson cohomology.* II, Amer. J. Math. **97** (1975), 101–123.

[Wu_1] U. Würgler, *On products in a family of cohomology theories associated to the invariant prime ideals of* $\pi_*(BP)$, Comment. Math. Helv. **52** (1977), 457–481.

[Wu_2] ———, *On the relation of Morava K-theories to Brown-Peterson homology,* Topology and Algebra, Proc. Colloq in Honor of B. Eckmann (Zurich 1977), 1978, pp. 269–280.

A splitting theorem for certain cohomology theories associated to $BP^*(-)$, Manuscripta Math. **29** (1979), 93–111.

[Zb] A. Zabrodsky, *Hopf spaces,* North-Holland Math. Studies, vol. 22; Notas de Matematica, no. 59, North-Holland, Amsterdam, 1976.

[Zh] R. S. Zahler, *Fringe families in stable homotopy of spheres,* Trans. Amer. Math. Soc. **224** (1976), 243–253.

ISBN 978-0-8218-1699-8

CBMS/48