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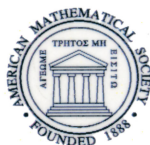
CBMS

Regional Conference Series in Mathematics

Number 49

Homology and Dynamical Systems

John M. Franks



American Mathematical Society
with support from the
National Science Foundation



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Preface

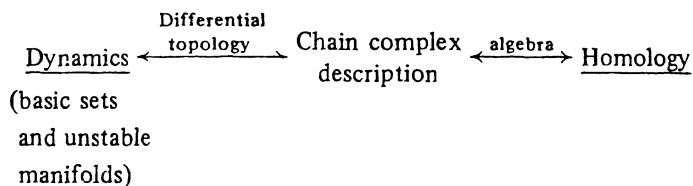
The qualitative study of the solutions to ordinary differential equations has had a long and varied history. In recent years much attention has been paid to the connections between the theory of these smooth “dynamical systems” and two other areas of mathematics: ergodic theory and statistical mechanics on the one hand and algebraic and differential topology on the other.

It is the relationships between dynamical systems and topology which these lectures address. This particular part of dynamical systems dates from the work of Poincaré, especially his beautiful theorem equating the Euler characteristic of a surface with the sum of the indices of rest points of a flow on the surface.

In this century the most important contributions to this area of investigation have been made by Marston Morse and Steve Smale. It is not possible to survey all their contributions in a meaningful way in this brief introduction. However, the importance of their contributions can perhaps be gauged by the number of times their names occur in the chapter titles of these lectures. It would be remiss however not to mention the special importance of Smale’s paper *Differentiable dynamical Systems* [S1], not only for investigations of the type we consider here, but all qualitative investigation of smooth dynamical systems. In addition to proving important new results, this article had a major influence on the direction of the whole field of dynamical systems. Two influences merit special mention. First it emphasized classifying dynamical systems according to the complexity of their qualitative dynamical behavior, rather than, for example, the more traditional way of classification by complexity of the algebraic form of the differential equation. Secondly Smale drew attention to structurally stable systems as particularly worthy of investigation and conjectured a characterization of them, which has subsequently been proven correct in many cases.

The relationship between qualitative dynamics and topology is much too large an area to consider fruitfully in its entirety within the framework of these lectures and accordingly we will narrow our attention to a collection of results with a particularly homological flavor.

The theme of these lectures is illustrated in the following diagram.



The left-hand arrow indicates that techniques of differential topology provide a connection between unstable manifolds (which we will see often form a cell decomposition) and a homological description of the system at the chain complex level. The right-hand arrow represents many algebraic results relating chain complex behavior with corresponding homological behavior. The bidirectionality of the arrows reflects the need, given one set of data, to understand what are the possibilities for the corresponding data at the other end of the arrow. The ultimate aim, of course, is to answer questions like, “In a given homological configuration, what kinds of dynamics can occur?” or “Given a dynamical configuration what are its homological implications?” This underlying theme recurs throughout these notes in many different guises, applied to many different classes of flows and diffeomorphisms.

In choosing the topics of these lectures compatibility with this theme has been my first criterion. In addition I have tried not to overlap too much with previous sets of lectures [B2, C, Gu, Mar, New, Sh], all of which deal with dynamical systems. This consideration and the unfortunate realization that I cannot include everything has led me to omit, for example, any discussion of the entropy conjecture, which has an unquestionable right to be in any treatise on homology and dynamical systems. In defense of its omission here I can only refer the reader to Chapter 5 of [B2].

Many conversations with colleagues too numerous to mention have been invaluable during the preparation of these lectures. Special thanks, however, are due to Steve Batterson, who is responsible for the existence of these lectures, as well as some of the theorems in them. I also wish to thank George Francis for drawing the illustrations to Appendix B.

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