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Preface

The qualitative study of the solutions to ordinary differential equations has had a long and varied history. In recent years much attention has been paid to the connections between the theory of these smooth "dynamical systems" and two other areas of mathematics: ergodic theory and statistical mechanics on the one hand and algebraic and differential topology on the other.

It is the relationships between dynamical systems and topology which these lectures address. This particular part of dynamical systems dates from the work of Poincaré, especially his beautiful theorem equating the Euler characteristic of a surface with the sum of the indices of rest points of a flow on the surface.

In this century the most important contributions to this area of investigation have been made by Marston Morse and Steve Smale. It is not possible to survey all their contributions in a meaningful way in this brief introduction. However, the importance of their contributions can perhaps be gauged by the number of times their names occur in the chapter titles of these lectures. It would be remiss however not to mention the special importance of Smale's paper *Differentiable dynamical Systems* [S1], not only for investigations of the type we consider here, but all qualitative investigation of smooth dynamical systems. In addition to proving important new results, this article had a major influence on the direction of the whole field of dynamical systems. Two influences merit special mention. First it emphasized classifying dynamical systems according to the complexity of their qualitative dynamical behavior, rather than, for example, the more traditional way of classification by complexity of the algebraic form of the differential equation. Secondly Smale drew attention to structurally stable systems as particularly worthy of investigation and conjectured a characterization of them, which has subsequently been proven correct in many cases.

The relationship between qualitative dynamics and topology is much too large an area to consider fruitfully in its entirety within the framework of these lectures and accordingly we will narrow our attention to a collection of results with a particularly homological flavor.

The theme of these lectures is illustrated in the following diagram.

$$\underline{\text{Dynamics}} \xleftrightarrow{\substack{\text{Differential}\\ \text{topology}}} \begin{array}{c}\text{Chain complex}\\ \text{description}\end{array} \xleftrightarrow{\text{algebra}} \underline{\text{Homology}}$$

(basic sets
and unstable
manifolds)

The left-hand arrow indicates that techniques of differential topology provide a connection between unstable manifolds (which we will see often form a cell decomposition) and a homological description of the system at the chain complex level. The right-hand arrow represents many algebraic results relating chain complex behavior with corresponding homological behavior. The bidirectionality of the arrows reflects the need, given one set of data, to understand what are the possibilities for the corresponding data at the other end of the arrow. The ultimate aim, of course, is to answer questions like, "In a given homological configuration, what kinds of dynamics can occur?" or "Given a dynamical configuration what are its homological implications?" This underlying theme recurs throughout these notes in many different guises, applied to many different classes of flows and diffeomorphisms.

In choosing the topics of these lectures compatibility with this theme has been my first criterion. In addition I have tried not to overlap too much with previous sets of lectures [**B2**, **C**, **Gu**, **Mar**, **New**, **Sh**], all of which deal with dynamical systems. This consideration and the unfortunate realization that I cannot include everything has led me to omit, for example, any discussion of the entropy conjecture, which has an unquestionable right to be in any treatise on homology and dynamical systems. In defense of its omission here I can only refer the reader to Chapter 5 of [**B2**].

Many conversations with colleagues too numerous to mention have been invaluable during the preparation of these lectures. Special thanks, however, are due to Steve Batterson, who is responsible for the existence of these lectures, as well as some of the theorems in them. I also wish to thank George Francis for drawing the illustrations to Appendix B.

References

[AM] M. Artin and B. Mazur, *On periodic points*, Ann. of Math. **81** (1969), 82–99.

[As1] D. Asimov, *Round handles and non-singular Morse-Smale flows*, Ann. of Math. **102** (1979), 41–54.

[As2] ———, *Homotopy of non-singular vector fields to structurally stable ones*, Ann. of Math. **102** (1979), 55–69.

[As3] ———, *Flaccidity of geometric index for non-singular vector fields*, Comment. Math. Helv. **52** (1977), 145–160.

[Bass1] H. Bass, *Introduction to some methods of algebraic K-theory*, C.B.M.S. Regional Conf. Series in Math., no. 20, Amer. Math. Soc., Providence, R. I., 1973.

[Bass2] ———, *The group SSF*, handwritten notes, 1979.

[Ba1] Steve Batterson, *Constructing Smale diffeomorphisms on compact surfaces*, Trans. Amer. Math. Soc. **257** (1980), 237–245.

[Ba2] ———, *The dynamics of Morse-Smale diffeomorphisms on the torus*, Trans. Amer. Math. Soc. **256** (1979), 395–403.

[Ba3] ———, *Orientation reversing Morse-Smale diffeomorphisms on the torus*, Trans. Amer. Math. Soc. (to appear).

[Bl-F] Paul Blanchard and John Franks, *The dynamical complexity of orientation reversing homeomorphisms of surfaces*, Invent. Math. **62** (1980), 333–339.

[Bl-F2] ———, *An obstruction to the existence of certain dynamics in surface diffeomorphisms*, Ergodic Theory Dynamical Systems **1** (1981), 255–260.

[BW] J. Birman and R. F. Williams, *Knotted periodic orbits in dynamical systems*. I, II (to appear).

[B1] R. Bowen, *Equilibrium states and the ergodic theory of Anosov diffeomorphisms*, Lecture Notes in Math., vol. 470, Springer-Verlag, Berlin and New York, 1975.

[B2] ———, *On Axiom A diffeomorphisms*, C.B.M.S. Regional Conf. Series in Math., no. 38, Amer. Math. Soc., Providence, R. I., 1978.

[B3] ———, *Entropy versus homology for certain diffeomorphisms*, Topology **13** (1974), 61–67.

[B4] ———, *One-dimensional hyperbolic sets for flows*, J. Differential Equations **12** (1972), 173–179.

[B5] ———, *Topological entropy and Axiom A*, Proc. Sympos. Pure. Math., vol. 14, Amer. Math. Soc., Providence, R. I., 1970, pp. 23–41.

[BF] R. Bowen and J. Franks, *Homology for zero dimensional basic sets*, Ann. of Math. **106** (1977), 73–92.

[BL] R. Bowen and O. Lanford, *Zeta functions of restrictions of the shift transformation*, Proc. Sympos. Pure Math., vol. 14, Amer. Math. Soc., Providence, R. I., 1970, pp. 43–49.

[C] Charles Conley, *Isolated invariant sets and the Morse index*, C.B.M.S. Regional Conf. Series in Math., no. 38, Amer. Math. Soc., Providence, R. I., 1978.

[D1] A. Dold, *Lectures on algebraic topology*, Springer-Verlag, Berlin, 1972.

[D2] ———, *Fixed point index and fixed point theorem for Euclidean neighborhood retracts*, Topology **4** (1965), 1–8.

[F-S] J. Franke and J. Selgrade, *Hyperbolicity and chain recurrence*, J. Differential Equations **26** (1977), 27–36.

[F1] J. Franks, *Constructing structurally stable diffeomorphisms*, Ann. of Math. **105** (1977), 343–359.

[F2] ———, *A reduced zeta function for diffeomorphisms*, Amer. J. Math. **100** (1978), 217–243.

[F3] ———, *Morse inequalities for zeta functions*, Ann. of Math. **102** (1975), 143–157.

[F4] ———, *The periodic structure of non-singular Morse-Smale flows*, Comment. Math. Helv. (1978), 279–294.

[F5] ———, *Morse-Smale flows and homotopy theory*, Topology **18** (1979), 199–215.

[F6] ———, *Knots, links and symbolic dynamics*, Ann. of Math. **113** (1981), 529–552.

[F7] ———, *Non-singular flows on S^3 with hyperbolic chain recurrent set*, Rocky Mountain J. Math. **7** (1977), 539–546.

[FN] J. Franks and C. Narasimhan, *The periodic behavior of Morse-Smale diffeomorphisms*, Invent. Math. **48** (1978), 279–292.

[F-Sh] J. Franks and M. Shub, *The existence of Morse-Smale diffeomorphisms*, Topology **20** (1981), 273–290.

[Fr1] D. Fried, *Cross-sections to flows*, Ph.D. thesis, University of California, Berkeley, 1976.

[Fr2] ———, *Flow equivalence, hyperbolic systems, and a new zeta function for flows* (to appear).

[Fr3] ———, *Subshifts on surfaces*, preprint.

[G] F. R. Gantmacher, *Theory of matrices*. II, Chelsea, New York, 1960.

[Gu] John Guckenheimer, *Bifurcations of dynamical systems*, Dynamical Systems, C.I.M.E. Lectures (Bressanone, Italy, 1978), Birkhauser, Basel, 1980, pp. 115–232.

[H] Mike Handel, *The entropy of orientation reversing homeomorphisms of surfaces*, preprint.

[HPPS] M. Hirsch, J. Palis, C. Pugh and M. Shub, *Neighborhoods of hyperbolic sets*, Invent. Math. **9** (1970), 121–134.

[H-P] M. Hirsch and C. Pugh, Proc. Sympos. Pure Math., vol. 14, Amer. Math. Soc., Providence, R.I., 1970, pp. 133–163.

[L] S. Lang, *Algebra*, Addison-Wesley, Reading, Mass., 1965.

[Ma] R. Mañe, untitled preprint.

[Mal] M. Maller, *Fitted diffeomorphisms of non-simply connected manifolds*, Topology 19 (1980), 395–410.

[Mar] L. Markus, *Lectures in differentiable dynamics*, C.B.M.S. Regional Conf. Series in Math. vol. 3, Amer. Math. Soc., Providence, R. I., 1971, (revised 1980).

[M1] J. Milnor, *Lectures on the H-cobordism theorem*, Princeton Univ. Press, Princeton, N. J., 1965.

[M2] ———, *An introduction to algebraic K-theory*, Ann. of Math. Studies, no. 72, Princeton, N. J., 1971.

[M3] ———, *Infinite cyclic coverings*, Conf. Topology of Manifolds (Michigan State, 1967), Prindle, Weber and Schmidt, Boston, Mass., 1968, pp. 115–133.

[M4] ———, *Topology from the differentiable viewpoint*, University Press of Virginia, Charlottesville, Va., 1965.

[Mo] J. Morgan, *Non-singular Morse-Smale flows in 3-dimensional manifolds*, Topology 18 (1978), 41–53; Errata.

[N] Carolynn Narasimhan, *The periodic behavior of Morse-Smale diffeomorphisms on compact surfaces*, Trans. Amer. Math. Soc. 248 (1979), 145–169.

[Ne] M. Newman, *Integral matrices*, Academic Press, New York, 1972.

[New] S. Newhouse, *Lectures on dynamical systems*, Dynamical Systems, C.I.M.E. Lectures (Bressanone, Italy, 1978), Birkauser, Basel, 1980, pp. 1–114.

[P] M. Peixoto, *Structural stability on two-dimensional manifolds*, Topology 1 (1962), 101–120.

[Pa-S] W. Parry and D. Sullivan, *A topological invariant of flows on one-dimensional spaces*, Topology 14 (1975), 297–299.

[P-S] C. Pugh and M. Shub, *The Ω-stability theorem for flows*, Invent. Math. 11 (1970), 150–158.

[Ro] J. Robbin, *A structural stability theorem*, Ann. of Math. 94 (1971), 447–493.

[R1] C. Robinson, *Structural stability of C^1 flows*, Dynamical Systems (Warwick, 1974), Lecture Notes in Math., vol. 468, Springer-Verlag, Berlin and New York, 1975, pp. 262–277.

[R2] ———, *Structural stability for C^1 diffeomorphisms*, J. Differential Equations 22 (1976), 28–73.

[S1] S. Smale, *Differentiable dynamical systems*, Bull. Amer. Math. Soc. 73 (1967), 797–817.

[S2] ———, *The Ω-stability theorem*, Proc. Sympos. Pure Math., vol. 14, Amer. Math. Soc., Providence, R. I., 1970, pp. 289–297.

[S3] ———, *On gradient dynamical systems*, Ann. of Math. 74 (1961), 199–206.

[S4] ———, *Diffeomorphisms with many periodic points*, Differential and Combinatorial Topology (S. Cairns, ed.) Princeton Univ. Press, Princeton, N. J., 1965.

[S5] S. Smale, *Notes on differentiable dynamical systems,* Proc. Sympos. Pure Math., vol. 14, Amer. Math. Soc., Providence, R. I., 1970, pp. 277–287.

[S6] ———, *Morse inequalities for a dynamical system,* Bull. Amer. Math. Soc. **66** (1960), 43–49.

[Sh] M. Shub, *Stabilité globale des systèmes dynamique,* Astérisque **56** (1978).

[SS] M. Shub and D. Sullivan, *Homology and dynamical systems,* Topology **14** (1975), 109–132.

[T] G. Torres, *On the Alexander polynomial,* Ann. of Math. **57** (1953), 57–89.

[W] W. Wilson, *Smoothing derivatives of functions and applications,* Trans. Amer. Math. Soc. **139** (1969), 413–428.

[Wm1] R. Williams, *Classification of subshifts of finite type,* Ann. of Math. **98** (1973), 120–153; Errata **99** (1974), 380–381.

[Wm2] ———, *The zeta function of an attractor,* Conf. Topology of Manifolds (Michigan State, 1967) Prindle, Weber and Schmidt, Boston, Mass., 1968.

[Wm3] ———, *The structure of Lorenz attractors,* Inst. Hautes Étude Sci. Publ. Math. **50** (1979), 73–100.

[Wh] J. H. C. Whitehead, *The mathematical works of J. H. C. Whitehead,* vol. 3, Pergamon Press, Oxford, 1962.

[Z] C. Zeeman, *Morse inequalities for diffeomorphisms with shoes and flows with solenoids,* Dynamical Systems (Warwick, 1974), Lecture Notes in Math., vol. 468, Springer-Verlag, Berlin and New York, 1975, pp. 44–47.

Added References

[deR] K. deRezende, *Smale flows on the three sphere,* Trans. Amer. Math. Soc. **303** (1987), 283-310.

[F8] J. Franks, *Flow equivalence of subshifts of finite type,* Ergodic Theory and Dynamical Systems, **4** (1984), 53-66.

[F9] J. Franks, *Symbolic Dynamics in Flows on Three-Manifolds,* Trans. Amer. Math. Soc. **279** (1983), 231-236.

[F10] J. Franks, *Nonsingular Smale flows on S^3,* Topology, **24** (1985), 265-282.

[F11] J. Franks, *Period Doubling and the Lefschetz Formula,* Trans. Amer. Math. Soc. **287** (1985), 275-283.

[FW] J. Franks and R. Williams, *Entropy and Knots,* Trans. Amer. Math. Soc. **291** (1985), 241-253.

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