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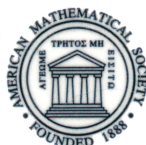
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Structure of Factors and Automorphism Groups

Masamichi Takesaki



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**STRUCTURE OF FACTORS
AND
AUTOMORPHISM GROUPS**

by
MASAMICHI TAKESAKI

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Contents

Introduction.....	1
Chapter I. Noncommutative Integration.....	3
Section 1. Left Hilbert algebras.....	4
Section 2. Weights and Representations.....	12
Section 3. Left Hilbert algebras and weights.....	14
Section 4. Weights and modular automorphism groups.....	16
Section 5. The centralizer of a weight.....	19
Section 6. Connes' theorem of Radon-Nikodym type.....	22
Section 7. Standard form of a von Neumann algebra.....	29
Chapter II. General Theory of Crossed Products and Duality.....	33
Section 1. Crossed products.....	34
Section 2. Duality for crossed products.....	37
Section 3. Arveson-Connes spectral theory.....	40
Section 4. Construction of covariant systems.....	45
Section 5. Comparison of cocycles.....	50
Section 6. Galois correspondence. Abelian case.....	54
Section 7. Galois correspondence. Noncommutative case.....	57
Chapter III. Structure of Factors of Type III.....	59
Section 1. Crossed product decomposition of von Neumann algebras of type III.....	59
Section 2. Factors of type III_λ , $0 < \lambda < 1$	65
Section 3. Factors of type III_0	67
Section 4. Factors associated with ergodic transformation groups.....	70
Section 5. Flow of weights.....	73
Chapter IV. Connes' Theory of Injective Factors and Automorphisms.....	77
Section 1. Centralizing sequences.....	77
Section 2. Noncommutative Følner's condition.....	82
Section 3. Characterizations of approximately finite dimensional factors.....	87
Section 4. Outer conjugacy of automorphisms of strongly stable factors.....	92
Section 5. Injective factors of type III_λ , $0 \leq \lambda < 1$	97
References.....	101

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References

The amount of literature on the subject has been increasing rapidly. It is simply impossible to list all papers. Many important papers are not cited here. A good source for literature hunting is the C^* -News, CPT/CNRS, University of Luminy, France.

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