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## Orderings, Valuations and Quadratic Forms

T. Y. Lam



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## Preface for the Second Printing

In the twelve years of its existence, the enthusiastic response to “Orderings, Valuations and Quadratic Forms” from the mathematical community has been most gratifying (and has in every way exceeded my own expectations). Now that the “pangs of birth” of the book are far behind me, I look back with satisfaction on the whole project, and recall with great pleasure that hot summer week in 1981 when I gave my ten lectures at Carleton, which eventually evolved into this book.

Today, in spite of (or perhaps due to?) the many more papers written on the subject, the need of this modest volume as a handy reference for the basic connections between orderings, valuations and quadratic forms seemed undiminished. I am thus very grateful to the American Mathematical Society for undertaking a second printing of this book to keep it in world-wide circulation. Taking advantage of this new printing, I have corrected a few typographical errors pointed out by my readers; otherwise, the main text has remained the same. Feedback and comments from my future readers are of course welcome, and may be sent to my e-mail address “lam@math.berkeley.edu”.

T. Y. L.

Berkeley, California  
January, 1996

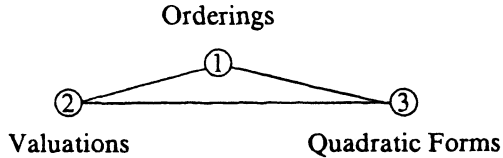


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## Introduction

These notes were developed from the ten lectures I gave at the CBMS Regional Conference held at Carleton College, Northfield, Minnesota in August, 1981. In the lectures, I covered essentially the equivalent of the first eight sections, but did not have time to go into the other interesting parts of the subject. In writing up these notes for publication, I have, therefore, added ten more sections of material, so that we can present to the reader a more comprehensive coverage of the subject matter at hand.

As is clear from the title, we deal in these notes with the following triumvirate of topics



in the category of formally real fields. There is perhaps one distinctive feature in our treatment: our main goal is not so much to develop each of these topics for its own sake, but rather, we lay special emphasis on the remarkable relations and connections which exist among the three. The connection between ① and ② is provided by the fact that, to any ordering  $P$  on a field  $F$ , we can associate a natural valuation on  $F$ , whose valuation ring consists of elements of  $F$  which are bounded by some rational numbers with respect to  $P$ : this basic construction can be traced back to the work of H. Hahn, Artin-Schreier, R. Baer and W. Krull. The connection between ② and ③, on the other hand, originated with the work of T. A. Springer. In his 1955 paper, Springer studied the behavior of quadratic forms over a field  $F$  equipped with a nondyadic complete discrete valuation, and determined the structure of the Witt ring of  $F$  via the notion of residue forms. Springer's work can be easily adapted to the case of nondyadic Henselian valuated fields, and the general case of a valuated field can be treated by passing to its Henselization. Finally, we come to the connection between ① and ③. This connection is provided by the important notion of a signature: given any  $F$ -quadratic form  $\varphi$ , and any ordering  $P$  on  $F$ , we can define  $\text{sgn}_P \varphi$  (the signature of  $\varphi$  with respect to  $P$ ) in the same way J. J. Sylvester defined the signature of a quadratic form over the real numbers. For a formally real field  $F$ , the total signature of  $\varphi$  (i.e. the signatures of  $\varphi$  with respect to the various orderings on  $F$ ) turns out to determine the Witt class of  $\varphi$  "up to torsion": this is the celebrated

**Local-Global Principle of A. Pfister.** This principle suggests that it may be useful to quotient out the torsion ideal in the Witt ring and pass to the so-called reduced Witt ring: here, we get an allied “reduced theory” of quadratic forms. Reduced forms are classified by their total signatures, so their behavior is governed by the structure of the space of orderings of the ground field.

The remarkable connections between ①, ② and ③ above and their ramifications have been the object of an intensive study in the recent literature. New tools have been developed and many interesting new results have been obtained, so there is now a fairly substantial body of knowledge revolving around the themes of orderings, valuations and quadratic forms. The present notes grew out of my modest aim to understand these themes and to organize them into a coherent structure. The presentation in these notes is largely self-contained, and does not presuppose any advanced knowledge of orderings or of quadratic forms. The only prerequisite is perhaps a good working knowledge of general valuation theory, and some familiarity with the basic notions and terminology of quadratic form theory as contained, for instance, in the first two chapters of my Benjamin book. The present notes may therefore be read either as a sequel to my Benjamin book, or as an independent introduction to ordered fields and reduced quadratic forms using valuation-theoretic techniques.

While we have tried to touch upon a broad spectrum of topics in these notes, no attempt is made at completeness. In order to keep the size of these notes within bounds, we have, in fact, left out some of the more advanced or more specialized topics. These include, for instance, Harrison’s theory of preprimes in rings and fields, Marshall’s theory of abstract spaces of orderings and their classification, Bröcker’s counting formula for the number of orderings on a field as well as his global stability index formula, the Bröcker-Prestel technique of intersecting Henselian fields and their use of the  $v$ -topology approximation theorem, the Berman-Cordes-Ware treatment of basic elements and the radical, etc. Many of these further topics are indicated in the historical notes at the end of the sections, so the interested reader should have no difficulty in tracking down the appropriate original sources in the literature.

These notes are entirely expository in nature, so the debt they owe to the work of other authors should be self-evident. I take this opportunity to thank all those from whose work my exposition has drawn. Portions of the material in these notes have been presented at various times by visiting participants in the Berkeley Seminar on Quadratic Forms in the period 1976-1981. I hereby thank all past participants in this seminar for the valuable contributions they have made. I am also greatly indebted to Eberhard Becker, Bill Jacob, David Leep, Alex Prestel, Alex Rosenberg, Dan Shapiro and Adrian Wadsworth who critically read various parts of the manuscript, caught many slips and made numerous valuable suggestions for improvements. It is a special pleasure to thank Steve Galovich who, as director of the Regional Conference at Carleton, helped create a particularly pleasant atmosphere in the Conference week for all the participants. In the ten

months following the Conference, Steve gave his time generously beyond the call of duty to help me in the preparation of these notes, and Barbara Jenkins cheerfully typed the whole manuscript. Without their help, the publication of these notes would no doubt have been greatly delayed.

Last but not least, I thank my wife Chee-King who accepted on faith the usefulness of this work and quietly tolerated my emotional fits in the agonizing months of its completion. In appreciation, I would like to dedicate this work to her.

Berkeley, California  
August, 1982

T.Y.L.

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## References

(The following contains all papers referred to in the text, plus a few others which are included because of their relevance to the general themes of these notes. However, our list is by no means complete; for more references on ordered fields, see the bibliography of my survey [98].)

1. J.-K. Arason, R. Elman and B. Jacob, *Graded Witt rings and Galois cohomology* (in preparation).
2. E. Artin, *Über die Zerlegung definiter Funktionen in Quadrate*, Abh. Math. Sem. Univ. Hamburg 5 (1927), 100–115.
3. E. Artin and O. Schreier, *Algebraische Konstruktion reeller Körper*, Abh. Math. Sem. Univ. Hamburg 5 (1927), 85–99.
4. \_\_\_\_\_, *Eine Kennzeichnung der reell abgeschlossenen Körper*, Abh. Math. Sem. Univ. Hamburg 5 (1927), 225–231.
5. J. Ax, *A metamathematical approach to some problems in number theory*, Proc. Sympos. Pure Math., Vol. 20, Amer. Math. Soc., Providence, R. I., 1969, pp. 161–190.
6. F. Bachmann, *Aufbau der Geometrie aus dem Spiegelungsbegriff*, Springer-Verlag, Berlin and New York, 1959.
7. R. Baer, *Über nicht-archimedisch geordnete Körper*, Sitz. Ber. der Heidelberger Akad. Abh., 1927, pp. 3–13.
8. \_\_\_\_\_, *Linear algebra and projective geometry*, Academic Press, New York, 1952.
9. \_\_\_\_\_, *Dichte, Archimedizität und Stårheit geordneter Körper*, Math. Ann. 188 (1970), 165–205.
10. S. A. Basarab, *Toward a general theory of formally  $v$ -adic fields*, Manuscripta Math. 30 (1980), 279–327.
11. E. Becker, *Euklidische Körper und euklidische Hüllen von Körpern*, J. Reine Angew. Math. 268/269 (1974), 41–52.
12. \_\_\_\_\_, *Hereditarily Pythagorean fields and orderings of higher level*, IMPA Lecture Notes, No. 29, Rio de Janeiro, 1978.
13. \_\_\_\_\_, *Partial orders on a field and valuation rings*, Comm. Algebra 7 (1979), 1933–1976.
14. \_\_\_\_\_, *Summen  $n$ -ter Potenzen in Körpern*, J. Reine Angew. Math. 307/308 (1979), 8–30.
15. \_\_\_\_\_, *Local global theorems for diagonal forms*, J. Reine Angew. Math. 318 (1980), 36–50.
16. \_\_\_\_\_, *Valuations and real places in the theory of formally real fields*, in Géométrie, Algèbrique Réelle et Formes Quadratiques, Proceedings of Conference in Rennes, Lecture Notes in Math., Vol. 959, Springer-Verlag, Berlin and New York, 1982, pp. 1–40.
17. \_\_\_\_\_, *The real holomorphy ring and sums of  $2n$ -th powers*, in Géométrie, Algèbrique Réelle et Formes Quadratiques, Proceedings of Conference in Rennes, Lecture Notes in Math., Vol. 959, Springer-Verlag, Berlin and New York, 1982, pp. 139–181.
18. E. Becker and L. Bröcker, *On the description of the reduced Witt ring*, J. Algebra 52 (1978), 328–346.
19. E. Becker, J. Harman and A. Rosenberg, *Signatures of fields and extension theory*, J. Reine Angew. Math. 330 (1982), 53–75.
20. E. Becker and E. Köpping, *Reduzierte quadratische Formen und Semiordnungen reeller Körper*, Abh. Math. Sem. Univ. Hamburg 46 (1977), 143–177.
21. E. Becker and A. Rosenberg, *Reduced forms and reduced Witt rings of higher level* (to appear).
22. E. Becker and N. Schwartz, *Zum Darstellungssatz von Kadison-Dubois*, to appear in Archiv Math.

23. L. Berman, *The Kaplansky radical and values of binary quadratic forms over fields*, Dissertation, Univ. of California, Berkeley, Calif., 1978.
24. L. Berman, C. Cordes and R. Ware, *Quadratic forms, rigid elements and power series fields*, J. Algebra **66** (1980), 123–133.
25. L. Bröcker, *Über eine Klasse pythagoreischer Körper*, Arch. Math. **23** (1972), 405–407.
26. ———, *Zur Theorie der quadratischen Formen über formal reellen Körpern*, Math. Ann. **210** (1974), 233–256.
27. ———, *Characterization of fans and hereditarily pythagorean fields*, Math. Z. **151** (1976), 149–163.
28. ———, *Über die Anzahl der Anordnungen eines kommutativen Körpers*, Arch. Math. **29** (1977), 458–464.
29. ———, *Positivbereiche in kommutativen Ringen*, Abh. Math. Sem. Univ. Hamburg (to appear).
30. ———, *Real spectra and distributions of signatures*, in Géométrie, Algèbre Réelle et Formes Quadratiques, Proceedings of Conference in Rennes, Lecture Notes in Math., Springer-Verlag, Berlin and New York, Vol. 959, 1982, pp. 249–272.
31. R. Brown, *Real places and ordered fields*, Rocky Mountain J. Math. **1** (1971), 633–636.
32. ———, *An approximation theorem for extended prime spots*, Canad. J. Math. **24** (1972), 167–184.
33. ———, *Superpythagorean fields*, J. Algebra **42** (1976), 483–494.
34. ———, *The reduced Witt ring of a formally real field*, Trans. Amer. Math. Soc. **230** (1977), 257–292.
35. ———, *Real valued places on the function field of an algebraic curve*, Houston J. Math. **6** (1980), 227–243.
36. ———, *Powers of the augmentation ideal in the Witt ring of a field*, Publ. Math. Debrecen **28** (1981), 75–77.
37. R. Brown and M. Marshall, *The reduced theory of quadratic forms*, Rocky Mountain J. Math. **11** (1981), 161–175.
38. G. Brumfiel, *Partially ordered rings and semi-algebraic geometry*, Lecture Notes of the London Math. Soc., Vol. 37, Cambridge Univ. Press, Cambridge, 1979.
39. M. Coste and M.-F. Coste-Roy, *La topologie du spectre réel*, Ordered Fields and Real Algebraic Geometry (D. Dubois and T. Récio, editors), Contemporary Math., Vol. 8, Amer. Math. Soc., Providence, R. I., 1982, pp. 27–59.
40. T. Craven, *The topological space of orderings of a rational function field*, Duke Math. J. **41** (1974), 339–347.
41. ———, *The Boolean space of orderings of a field*, Trans. Amer. Math. Soc. **209** (1975), 225–235.
42. ———, *Existence of SAP extension fields*, Arch. Math. **29** (1977), 594–597.
43. ———, *Stability in Witt rings*, Trans. Amer. Math. Soc. **225** (1977), 227–242.
44. ———, *Characterizing reduced Witt rings*, J. Algebra **53** (1978), 68–77.
45. ———, *Characterizing reduced Witt rings. II*, Pacific J. Math. **80** (1979), 341–349.
46. ———, *Witt rings and orderings of skew fields*, J. Algebra **77** (1982), 74–96.
47. J. Diller and A. Dress, *Zur Galoistheorie pythagoreischer Körper*, Arch. Math. **16** (1965), 148–152.
48. A. Dress, *Lotschnittebenen mit halbierbarem rechten Winkel*, Arch. Math. **16** (1965), 388–392.
49. ———, *Ebenen über quadratisch perfekten Körpern*, Math. Z. **92** (1966), 19–29.
50. ———, *On orderings and valuations of fields*, Geom. Dedicata **6** (1977), 259–266.
51. D. Dubois, *A note on David Harrison's theory of preprimes*, Pacific J. Math. **21** (1967), 15–19.
52. ———, *Second note on David Harrison's theory of preprimes*, Pacific J. Math. **24** (1968), 57–68.
53. ———, *Infinite primes and ordered fields*, Dissertationes Math. (Warsaw) **69** (1970), 1–43.
54. D. Dubois and G. Efrogmson, *Algebraic theory of real varieties*, Studies and Essays presented to Yu-Why Chen on his 60th birthday, Taiwan University, 1970, pp. 107–135.
55. D. Dubois and T. Récio, *Order extensions and real algebraic geometry*, Ordered Fields and Real Algebraic Geometry (D. Dubois and T. Récio, editors), Contemporary Math., Vol. 8, Amer. Math. Soc., Providence, R. I., 1982, pp. 265–288.
56. W. Durfee, *Quadratic forms over fields with a valuation*, Bull. Amer. Math. Soc. **54** (1948), 338–351.

57. R. Elman, *Quadratic forms and the u-invariant*. III, Proc. Quadratic Form Conf. (G. Orzech, editor), Queen's Papers in Pure and Appl. Math., No. 46, Kingston, Ontario, 1976, pp. 422–444.
58. R. Elman and T. Y. Lam, *Pfister forms and K-theory of fields*, J. Algebra **23** (1972), 181–213.
59. ———, *Quadratic forms over formally real fields and pythagorean fields*, Amer. J. Math. **94** (1972), 1155–1194.
60. ———, *Quadratic forms under algebraic extensions*, Math. Ann. **219** (1976), 21–42.
61. R. Elman, T. Y. Lam and A. Prestel, *On some Hasse principles over formally real fields*, Math. Z. **134** (1973), 291–301.
62. R. Elman, T. Y. Lam and A. Wadsworth, *Orderings under field extensions*, J. Reine Angew. Math. **306** (1979), 7–27.
63. R. Elman, T. Y. Lam, A. Prestel and A. Wadsworth, *SAP pairs and SAP triples*, 1978 (unpublished).
64. R. Elman and A. Prestel, *Reduced stability of the Witt ring of a field and its pythagorean closure*, 1982, preprint.
65. O. Endler, *Valuation theory*, Springer-Verlag, Berlin and New York, 1972.
66. H. N. Gupta and A. Prestel, *On a class of Pasch-free euclidean planes*, Bull. Acad. Polon. Sci., Ser. Sci. Math. Astronom. Phys. **20** (1972), 17–23.
67. ———, *Triangular and Schwarz inequality in Pasch-free euclidean geometry*, Ibid. **20** (1972), 999–1003.
68. J. Harman, *Chains of higher level orderings*, Thesis, Univ. of California, Berkeley, Calif., 1980.
69. D. Harrison, *Finite and infinite primes for rings and fields*, Mem. Amer. Math. Soc. No. 68, Amer. Math. Soc., Providence, R. I., 1968.
70. D. Harrison and H. Warner, *Infinite primes for fields and completions*, Pacific J. Math. **45** (1973), 201–206.
71. S. Holland, *Orderings and square roots of \*-fields*, J. Algebra **46** (1977), 207–219.
72. ———, *\*-valuations and ordered \*-fields*, Trans. Amer. Math. Soc. **262** (1980), 219–243.
73. B. Jacob, *On the structure of pythagorean fields*, J. Algebra **68** (1981), 247–267.
74. ———, *Fans, valuations, and hereditarily pythagorean fields*, Pacific J. Math. **93** (1981), 95–105.
75. ———, *The Galois cohomology of pythagorean fields*, Invent. Math. **65** (1981), 97–113.
76. I. Kaplansky, *Fröhlich's local quadratic forms*, J. Reine Angew. Math. **239** (1969), 74–77.
77. J. Kleinstein and A. Rosenberg, *Signatures and semisignatures of abstract Witt rings and Witt rings of semilocal rings*, Canad. J. Math. **30** (1978), 872–895.
78. J. Kleinstein and A. Rosenberg, *Succinct and representational Witt rings*, Pacific J. Math. **86** (1980), 99–137.
79. M. Knebusch, *On the extension of real places*, Comment. Math. Helv. **48** (1973), 354–369.
80. ———, *Specialization of quadratic and symmetric bilinear forms, and a norm theorem*, Acta Arith. **24** (1973), 279–299.
81. ———, *Generalization of a theorem of Artin-Pfister to arbitrary semilocal rings and related topics*, J. Algebra **36** (1975), 46–67.
82. ———, *Symmetric bilinear forms over algebraic varieties*, Proc. Quadratic Form Conf. (G. Orzech, editor), Queen's Papers in Pure and Appl. Math., Vol. 46, Kingston, Ontario, 1976, pp. 103–283.
83. ———, *On algebraic curves over real closed fields*. I, Math. Z. **150** (1976), 49–70; Berichtigung: Math. Z. **155** (1977), 299.
84. ———, *On algebraic curves over real closed fields*. II, Math. Z. **151** (1976), 189–205.
85. ———, *Remarks on the paper "Equivalent topological properties of the space of signatures of a semilocal ring" by A. Rosenberg and R. Ware*, Publ. Math. Debrecen **24** (1977), 181–188.
86. ———, *Signatures, reelle Stellen und reduzierte quadratische Formen*, Jber. Deutsch. Math.-Verein. **82** (1980), 109–127.
87. ———, *On the local theory of signatures and reduced quadratic forms*, Abh. Math. Sem. Univ. Hamburg **51** (1981), 149–195.
88. M. Knebusch and M. Kolster, *Wittringe*. Der Regensburger Trichter, Band 14, Regensburg, Germany, 1978. (Also published by Braunschweig: Vieweg, 1982.)
89. M. Knebusch, A. Rosenberg and R. Ware, *Structure of Witt rings, quotients of abelian group rings, and orderings of fields*, Bull. Amer. Math. Soc. **77** (1971), 205–210.
90. ———, *Signatures on semilocal rings*, J. Algebra **26** (1973), 208–250.



91. M. Knebusch and M. Wright, *Bewertungen mit reeller Henselisierung*, J. Reine Angew. Math. **286/287** (1976), 314–321.
92. S. Kochen, *Integer valued rational functions over  $p$ -adic numbers, a  $p$ -adic analogue of the theory of real fields*, Proc. Sympos. Pure Math., Vol. 12, Number Theory, Amer. Math. Soc., Providence, R. I., 1969, pp. 57–73.
93. W. Krull, *Allgemeine Bewertungstheorie*, J. Reine Angew. Math. **167** (1931), 160–196.
94. M. Kula, *Ordered fields and quadratic forms over pythagorean fields*, Uniw. Ślaski w Katowicach Prace Nauk.-Prace Mat. **158** (1977), 13–21.
95. \_\_\_\_\_, *Fields with prescribed quadratic form schemes*, Math. Z. **167** (1979), 201–212.
96. T. Y. Lam, *The algebraic theory of quadratic forms*, Benjamin, Reading, Mass., 1973, (2nd Printing Rev., 1980).
97. \_\_\_\_\_, *Ten lectures on quadratic forms over fields*, Proc. Quadratic Form Conf. (G. Orzech, editor), Queen's Papers in Pure and Appl. Math., Vol. 46, Kingston, Ontario, 1976, pp. 1–102.
98. \_\_\_\_\_, *The theory of ordered fields*, Ring Theory and Algebra III (B. McDonald, editor), Lecture Notes in Pure and Applied Math., Vol. 55, Dekker, New York, 1980, 1–152.
99. \_\_\_\_\_, *An introduction to real algebra*, Lecture Notes, Sexta Escuela Latinoamericana de Matematicas. Oaxtepec, Mexico, 1982.
100. S. Lang, *The theory of real places*, Ann. of Math. **57** (1953), 378–391.
101. J. Leicht and F. Lorenz, *Die Primideale des Wittschen Ringes*, Invent. Math. **10** (1970), 82–88.
102. F. Lorenz, *Quadratische Formen über Körpern*, Lecture Notes in Math., Vol. 130, Springer-Verlag, Berlin and New York, 1970.
103. \_\_\_\_\_, *Quadratische Formen und die Artin-Schreiersche Theorie der formal reellen Körper*, Bull. Soc. Math. France Mém. **48** (1978), 61–73.
104. L. Mahé, *Signatures et composantes connexes*, Math. Ann. **260** (1982), 191–210.
105. M. Marshall, *A reduced theory of quadratic forms*, 1975, (unpublished notes).
106. \_\_\_\_\_, *Some local-global principles for formally real fields*, Canad. J. Math. **29** (1977), 606–614.
107. \_\_\_\_\_, *Classification of finite spaces of orderings*, Canad. J. Math. **31** (1979), 320–330.
108. \_\_\_\_\_, *Quotients and inverse limits of spaces of orderings*, Canad. J. Math. **31** (1979), 604–616.
109. \_\_\_\_\_, *The Witt ring of a space of orderings*, Trans. Amer. Math. Soc. **258** (1980), 505–521.
110. \_\_\_\_\_, *Spaces of orderings. IV*, Canad. J. Math. **32** (1980), 603–627.
111. \_\_\_\_\_, *Abstract Witt rings*, Queen's Papers in Pure and Appl. Math., Vol. 57, Kingston, Ontario, 1980.
112. C. Massaza, *Sulle valutazioni che inducono la topologia di un ordinamento non-archimedeo*, Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. **109** (1975), 343–359.
113. J. Merzel, *Quadratic forms over fields with finitely many orderings*, Ordered Fields and Real Algebraic Geometry (D. Dubois and T. Rézio, editors), Contemporary Math., Vol. 8, Amer. Math. Soc., Providence, R. I., 1982, pp. 185–229.
114. J. Milnor, *Algebraic K-theory and quadratic forms*, Invent. Math. **9** (1970), 318–344.
115. M. Nagata, *Some remarks on ordered fields*, Japan J. Math. **1** (1975), 1–4. (See also. Sem. Dubreil (1974/75), *Algèbre*, Exp. No. 26, Paris.)
116. W. Pejas, *Die Modelle des Hilbertschen Axiomensystems der absoluten Geometrie*, Math. Ann. **143** (1961), 212–235.
117. A. Pfister, *Quadratische Formen in beliebigen Körpern*, Invent. Math. **1** (1966), 116–132.
118. A. Prestel, *Quadratische Semi-Ordnungen und quadratische Formen*, Math. Z. **133** (1973), 319–342.
119. \_\_\_\_\_, *Euklidische Geometrie ohne das Axiom von Pasch*, Abh. Math. Sem. Univ. Hamburg **41** (1974), 82–109.
120. \_\_\_\_\_, *Lectures on formally real fields*, IMPA Lecture Notes, No. 22, Rio de Janeiro, 1975.
121. \_\_\_\_\_, *A local-global principle for quadratic forms*, Math. Z. **142** (1975), 91–95.
122. \_\_\_\_\_, *Local-global principles for quadratic forms over function fields*, Proc. Quadratic Form Conf. (G. Orzech, editor), Queen's Papers in Pure and Appl. Math., Vol. 46, Kingston, Ontario, 1976, pp. 595–612.
123. \_\_\_\_\_, *Remarks on the Pythagoras and Hasse number of real fields*, J. Reine Angew. Math. **303** (1978), 284–294.
124. \_\_\_\_\_, *Decidable theories of preordered fields*, Math. Ann. **258** (1982), 481–492.

125. A. Prestel and P. Roquette, *Lectures on formally  $p$ -adic fields*, IMPA Lecture Notes, Rio de Janeiro (to appear).
126. A. Prestel and R. Ware, *Almost isotropic quadratic forms*, J. London Math. Soc. **19** (1979), 241–244.
127. A. Prestel and M. Ziegler, *Erblich euklidische Körper*, J. Reine Angew. Math. **274/275** (1975), 196–205.
128. S. Priess-Crampe, *Zum Hahnschen Einbettungssatz für angeordnete Körper*, Arch. Math. **24** (1973), 607–614.
129. P. Ribenboim, *Théorie des valuations*, Presses Univ. Montréal, 1964.
130. \_\_\_\_\_, *On orderable fields*, Math. Nachr. **40** (1969), 343–355.
131. P. Roquette, *Bemerkungen zur Theorie der formal  $p$ -adischen Körper*, Wiss. Beiträge Martin-Luther-Univ. Halle-Wittenberg M3, 1971, pp. 177–193.
132. \_\_\_\_\_, *Principal ideal theorems for holomorphy rings in fields*, J. Reine Angew. Math. **262/263** (1973), 361–374.
133. \_\_\_\_\_, *On the Riemann  $p$ -space of a field. The  $p$ -adic analogue of Weierstrass' approximation theorem and related problems*, Abh. Math. Sem. Univ. Hamburg **97** (1978), 236–259.
134. A. Rosenberg and R. Ware, *Equivalent topological properties of the space of signatures of a semilocal ring*, Publ. Math. Debrecen **23** (1976), 283–289.
135. W. Scharlau, *Quadratic forms*, Queen's Papers in Pure and Appl. Math., Vol. 22, Kingston, Ontario, 1969.
136. H.-W. Schülting, *Über reelle Stellen eines Körpers und ihren Holomorphierung*, Dissertation, Universität Dortmund, 1979.
137. \_\_\_\_\_, *Über die Erzeugendenanzahl invertierbarer Ideale in Prüferingen*, Comm. Algebra **7** (1979), 1331–1349.
138. \_\_\_\_\_, *On real places of a field and their holomorphy ring*, Comm. Algebra **10** (1982), 1239–1284.
139. \_\_\_\_\_, *Real points and real places*, Ordered Fields and Real Algebraic Geometry (D. Dubois and T. Récio, editors), Contemporary Math., Vol. 8, Amer. Math. Soc., Providence, R. I., 1982, pp. 289–295.
140. \_\_\_\_\_, *Real holomorphy rings in real algebraic geometry*, in Géométrie Algébrique Réelle et Formes Quadratiques, Proceedings of Conference in Rennes, Lecture Notes in Math., Vol. 959, 1982, Springer-Verlag, Berlin and New York, pp. 433–442.
141. N. Schwartz, *Local stability and saturation in spaces of orderings*, Canad. J. Math. (to appear).
142. \_\_\_\_\_, *The strong topology on real algebraic varieties*, Ordered Fields and Real Algebraic Geometry (D. Dubois and T. Récio, editors), Contemporary Math., Vol. 8, Amer. Math. Soc., Providence, R. I., 1982, pp. 297–325.
143. \_\_\_\_\_, *Der Raum der Zusammenhangskomponenten einer reellen Varietät*, Geom. Dedicata (to appear).
144. J.-P. Serre, *Extensions de corps ordonnés*, C. R. Acad. Sci. Paris **229** (1949), 576–577.
145. D. Shapiro and T. Y. Lam, *The square class invariant for pythagorean fields*, Ordered Fields and Real Algebraic Geometry (D. Dubois and T. Récio, editors), Contemporary Math., Vol. 8, Amer. Math. Soc., Providence, R. I., 1982, pp. 327–340.
146. T. A. Springer, *Quadratic forms over fields with a discrete valuation*, Indag. Math. **17** (1955), 352–362.
147. R. Swan,  *$n$ -Generator ideals in Prüfer domains*, Pacific J. Math. (to appear).
148. L. Szczerba, *Independence of Pasch's Axiom*, Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys. **18** (1970), 491–498.
149. L. Szczerba and W. Szmielew, *On the euclidean geometry without the Pasch axiom*, Ibid. **18** (1970), 659–665.
150. W. Szmielew, *The Pasch axiom as a consequence of the circle axiom*, Ibid. **18** (1970), 751–758.
151. K. Szymiczek, *Quadratic forms over fields*, Dissertationes Math. (Rozprawy Mat.) **52** (1977), 1–63.
152. U.-P. Tietze, *Zur Theorie quadratischer Formen über Hensel-Körpern*, Arch. Math. **25** (1974), 144–150.
153. A. Tschimmel, *Diplomarbeit*, Universität Münster, 1976.
154. \_\_\_\_\_, *Über Anordnungsräume von Schiefkörpern*, Dissertation, Universität Münster, 1981.
155. L. van den Dries, *The model theory of fields*, Dissertation, Amsterdam, 1978.

156. R. von Chossy and S. Priess-Crampe, *Ordnungsverträgliche Bewertungen eines angeordneten Körpers*, Arch. Math. **26** (1975), 372–387.
157. A. Wadsworth, *p-Henselian fields: K-theory, Galois cohomology and graded Witt rings*, Pacific J. Math. (to appear).
158. R. Ware, *When are Witt rings group rings?* Pacific J. Math. **49** (1973), 279–284.
159. \_\_\_\_\_, *Extending orderings on formally real fields*, Arch. Math. **26** (1975), 611–614.
160. \_\_\_\_\_, *Hasse principles and the u-invariant over formally real fields*, Nagoya Math. J. **61** (1976), 117–125.
161. \_\_\_\_\_, *When are Witt rings group rings. II*, Pacific J. Math. **76** (1978), 541–564.
162. \_\_\_\_\_, *Valuation rings and rigid elements in fields*, Canad. J. Math. **33** (1981), 1338–1355.
163. E. Witt, *Theorie der quadratischen Formen in beliebigen Körpern*, J. Reine Angew. Math. **176** (1937), 31–44.
164. W. Scharlau, *Quadratische Formen und Galois-Cohomologie*, Invent. Math. **4** (1967), 238–264.

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