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Number 56

Dual Algebras with Applications to Invariant Subspaces and Dilation Theory

Hari Bercovici
Ciprian Foiaş
Carl Pearcy



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This book is dedicated to Paul R. Halmos and Béla Sz.-Nagy,
who have helped and inspired the authors in many ways
over a period of many years.

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Preface

This book is a slightly expanded and revised version of the lecture notes from the NSF/CBMS Regional Conference held in Tempe, Arizona in May, 1984, at which the third author was the principal lecturer. In the book we have tried to summarize some of the voluminous progress that has been made in the theory of dual algebras since the appearance in 1978 of Scott Brown's pioneering paper, which clearly showed the utility of this concept for studying the structure theory of bounded linear operators on Hilbert space. The aim of the book is to present an approach for studying (nonselfadjoint) dual algebras that allows one to obtain, in particular, results on invariant subspaces and dilation theory.

The book is put together as follows. Chapter I consists of preliminaries of a general nature concerning dual algebras. Most of Chapters II, III, IV, and X are taken from [6a], but Chapters III and VI contain some new material, especially Theorem 6.3 and the results leading up to it. Chapters IV and V are taken from [10], and Chapter VIII comes from [7a] and [6b]. Chapter VIII is a rewrite of part of [11] and [12a], and Chapter IX is taken from a version of [11a].

We wish to acknowledge our indebtedness to Constantin Apostol and Béla Sz.-Nagy, with whom we obtained many of the results to be found herein, and to Frank Gilfeather and the other members of the Mathematical Sciences Section of the National Science Foundation, whose support and encouragement have contributed greatly to our efforts.

Ann Arbor, Michigan
December 1984

Hari Bercovici
Ciprian Foiaş
Carl Pearcy

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Tempe, Arizona
May 15, 1984

Carl Percy

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