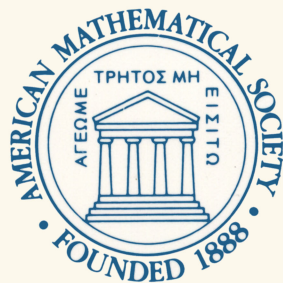


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number **60**



**Gilles Pisier**

# **FACTORIZATION OF LINEAR OPERATORS AND GEOMETRY OF BANACH SPACES**



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AND GEOMETRY OF BANACH SPACES**

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## Introduction

In 1956 Grothendieck published a fascinating paper entitled *Résumé de la théorie métrique des produits tensoriels topologiques*. This paper, which is now referred to as “the Résumé”, has had a considerable influence on the development of Banach space theory since 1968. It contained a general theory of tensor norms on tensor products of Banach spaces, described several operations to generate new tensor norms from some known ones, and studied the duality theory of these norms. But the highlight of the Résumé is a result that Grothendieck called “the fundamental theorem of the metric theory of tensor products” and which is now called Grothendieck’s theorem (or sometimes Grothendieck’s inequality). Among its many consequences, it implies that every bounded operator from  $L_\infty$  into  $L_1$  factors through  $L_2$ . This theorem remained practically unnoticed until 1968, when Lindenstrauss and Pełczyński revived it and gave a detailed proof (cf. [L-P]). Although there are now numerous simple proofs of this theorem (cf. e.g. Chapter 5), it remains a nontrivial result.

The aim of the present lecture notes is to describe the contributions made since 1968 in the directions opened by the Résumé. Although our title is very general, we will limitate ourselves to the work which is directly related to the questions raised in Grothendieck’s paper. The Résumé ends with a list of six problems with comments on each of them. Thanks to the considerable progress achieved in Banach space theory in the last 15 years, these problems are now all solved (except perhaps for the exact value of the Grothendieck constant), and these lecture notes will include the various results which led to their solution. These six problems are actually all linked together and related to several central questions. To summarize simply the contents of these notes, we might say that they revolve around the following questions: When does an operator  $u: X \rightarrow Y$  (between two Banach spaces) factor through a Hilbert space? For which spaces  $X, Y$  does this happen for all operators  $u$ ? We will examine the particular case of operators defined on a Banach lattice, a  $C^*$ -algebra, or the disc algebra and  $H^\infty$ .

The topics that we cover have many connections or applications outside Banach space theory, and we hope that they will have even more in the future. With this in mind, we have tried to make this material accessible to nonspecialists, so that our redaction is usually quite detailed and self-contained. For the same



reason, we have deliberately kept to a minimum the use of the duality theory via the trace, since we feel that this might turn off the readers who are not familiar with it. Nevertheless, we urge the readers who want to go deeper in the theory to get acquainted with the principles of this duality (cf. [P1 or Pe4]). We should mention that our restricted selection has left out several important topics. We refer to [P1] for the general theory of operator ideals which was developed by Pietsch and his school since the late sixties. The characterization of  $L_p$ -spaces (or subspaces of  $L_p$  or subspaces of quotients of  $L_p$ ) by operator theoretic properties is a major omission. For this, we refer the reader to the beautiful paper of Kwapien [Kw3] and to its references. Also, the factorization theorems of Maurey (and the important work of Rosenthal [R2]) are not included here; we refer the reader to [M1]. We do discuss, however, the general theory of type and cotype, but briefly and without proofs. We will be mainly concerned here with type 2 or cotype 2. In general, we have concentrated on the problem of factoring an operator through  $L_2$ , and we have left out the natural extensions for the factorization through  $L_p$ . In our exposition, we will come across most of the line of investigation which forms the so-called local theory of Banach spaces—i.e., the study of Banach spaces by finite-dimensional methods. We have tried to indicate in the references, as often as possible, the ramifications of this currently very active area.

Let us now review the contents of these notes. In Chapter 0, we introduce the projective and injective tensor products and the approximation property (in short A.P.). Among the six problems at the end of the Résumé, the first and most famous one was the approximation problem: Does every Banach space possess the A.P.? Enflo [E] gave a counterexample in 1972, which opened a new era in functional analysis.

In Chapter 0, we have insisted on the necessary distinction between nuclear operators and elements of the projective tensor product, which is essential in Chapter 10.

In Chapter 1, we present in detail the basic theory of  $p$ -summing operators and its first applications to Banach space theory: For every  $n$ -dimensional subspace  $E$  of a space  $X$ , there is a projection  $P: X \rightarrow E$  such that  $\|P\| \leq \sqrt{n}$  and an isomorphism  $T: l_2^n \rightarrow E$  such that  $\|T\| \|T^{-1}\| \leq \sqrt{n}$ .

In §c, we briefly introduce  $p$ -integral operators and some rudiments of duality theory, but this is not used in the sequel. We note in passing that the Radon-Nikodým property (which is crucial to compare integral and nuclear operators) is not discussed at all here; we refer the reader to [D-U] for this topic. In Chapter 2, we give the Lindenstrauss-Pelczyński criterion for an operator to factor through a Hilbert space. This can be viewed as an application of the Hahn-Banach theorem provided a certain duality theorem is explicited; we do this in §2.b. In Chapter 3, we introduce the notions of type and cotype and prove Kwapien's theorem that every space of type 2 and of cotype 2 is isomorphic to a Hilbert space. The theory of type and cotype provides a useful scale to measure how close a given space is

from a Hilbert space. We briefly review the main points of this theory in §3.3 (we use only the extreme cases of type 2 or cotype 2 in the sequel). In Chapter 4, we prove a factorization theorem which links Kwapien's theorem and Grothendieck's theorem: If  $X^*$  and  $Y$  are of cotype 2, then every approximable operator from  $X$  into  $Y$  factors through a Hilbert space. This result plays a crucial rôle in the construction of Chapter 10. As an application, in §4.b, we show that Sidon sets in the dual of a compact Abelian group  $G$  are characterized by the fact that they span a cotype 2 space in  $C(G)$ . This generalizes an earlier result of Varopoulos [V1]. In Chapter 5, we concentrate on Grothendieck's theorem, which we abbreviate G.T. Chapter 5 contains at least four proofs of that theorem. In §5.a, we briefly introduce  $\mathcal{L}_p$ -spaces (there is more information in §§8.b and 8.c). We are mainly concerned here with the cases  $p = 1$  and  $p = \infty$ . This allows us to state and prove G.T. in the framework of [L-P]: Every operator from an  $\mathcal{L}_1$  space into an  $\mathcal{L}_2$  space is 1-summing. This is proved in §5.c. In §5.b we give the (somewhat dual) formulation about operators defined on a  $C(K)$ -space or on an  $\mathcal{L}_\infty$ -space. We tried to give explicitly all the various forms in which the theorem can be used, and we distinguished carefully between the easy part (which we call the "little G.T.") and the more delicate part of this theorem. We first give a proof derived from the more "abstract" result of Chapter 4, but §5.d contains another proof, more direct and of independent interest.

In §5.3, we include Krivine's proof of G.T., which gives the best known upper bound for the constant  $K_G$ . In problem 3 in the *Résumé*, Grothendieck asked for the exact value of various constants (see 5.3 for details); this is the only problem which is not completely solved (but of course, it is probably the least important one!). In 5.f, we give a very quick proof of G.T., based on a property of the space  $H^1$ , due to Pełczyński and Wojtaszczyk.

In Chapter 6, we study the Banach spaces satisfying G.T., which we call G.T. spaces. We include several characterizations of these spaces, but we insist more on the a priori smaller class of G.T. spaces of cotype 2. The latter enjoys nicer stability properties and includes all the known examples of G.T. spaces. In 6.c, we show that if  $R$  is a hilbertian (or, more generally, a reflexive) subspace of  $L_1$ , then  $L_1/R$  is a G.T. space of cotype 2.

We come here to problem 5 in the *Résumé*. A stronger formulation of this problem was given by Lindenstrauss and Pełczyński, who asked whether the  $\mathcal{L}_1$ -spaces are the only spaces satisfying G.T. The above result of 6.c (due to Kisliakov and the author) gives a negative answer to this question (and a fortiori to problem 5) since the quotients  $L_1/R$  are never  $\mathcal{L}_1$ -spaces when  $R$  is a reflexive infinite-dimensional subspace of  $L_1$ .

It is rather easy to give concrete examples of hilbertian subspaces of  $L_1$ : for example, the span of the Rademacher functions. However, it is more delicate to produce "very large" such spaces. For this purpose, we present in Chapter 7 a method based on volume estimates which yields an orthogonal decomposition of  $l_1^{2^n}$  into two parts which are uniformly (with respect to  $n$ ) isomorphic to  $l_2^n$ . This

result originates in the work of Kašin, but the method was developed in [Sz2] and [S-T].

We also give in §8.g an infinite-dimensional version of this decomposition, obtained recently by Krivine.

In Chapter 8, we turn to Banach lattices and start by a reformulation of G.T. in this context. In 8.b, we introduce ultraproducts with several simple illustrative applications. Problem 2 in the Résumé asked whether a specific property (involving tensor norms) was always satisfied. This was answered negatively by Gordon-Lewis [G-L1]. Their paper showed that this property (now called the G.L. property) provides a useful criterion to decide whether or not a given space is isomorphic to a Banach lattice (or more generally to a space with l.u.st.). This is the subject of 8.c; in 8.d, we show that, for  $p \neq 2$ , the Schatten classes  $C_p$  do not have the G.L. property (cf. [G-L1, Sc]). Many more spaces without l.u.st. are now known. Moreover, one can construct, for any  $n$ , an  $n$ -dimensional space with l.u.st. constant greater than  $\delta\sqrt{n}$ , for some  $\delta > 0$  independent of  $n$ . This “worst possible” case can be exhibited in 8.e very quickly (following [F-K-P]), using Chapter 7. In §8.g, we show (following [L-P]) that an atomic Banach lattice which satisfies G.T. must be isomorphic to  $l_1(\Gamma)$  for some set  $\Gamma$ .

In Chapter 9, we present the  $C^*$ -algebraic version of G.T., as conjectured by Grothendieck. Here we mainly follow [Pi7] and Haagerup’s work [H1]. This was problem 4 in the Résumé. In §7.b, we discuss (without proofs) several applications of these results to the theory of derivations and representations of  $C^*$ -algebras (cf. [Bu, C1, C2, H2, H3]).

Finally, in Chapter 10, we construct (following [Pi10]) several Banach spaces  $X$  such that  $X \hat{\otimes} X = X \otimes X$ . This gives a negative solution to the sixth and last problem in the Résumé. Grothendieck conjectured there that this could happen only in the finite-dimensional case. The reader who has reached this point will be rewarded to find that all the results used in the construction have been included (with complete proofs) in the preceding chapters (mainly in Chapters 4, 7 and 6.c).

Each chapter is followed by a notes and references section where the reader will find the credits for the corresponding results, as well as some additional comments. In general, we give references in the text itself only for the statements which we quote without proof.

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## References

- [Be] S. Bellenot, *Uniformly complemented  $l_p^n$ 's in quasi-reflexive spaces*, Israel J. Math. **39** (1981), 234–246.
- [B-L] J. Bergh and J. Löfström, *Interpolation spaces*, Springer-Verlag, Berlin and New York, 1976.
- [Bl1] R. Blei, *A uniformity property for  $\Lambda(2)$  sets and inequality and applications*, Ark. Math. **17** (1979), 51–68.
- [Bl2] ———, *Multi dimensional extensions of the Grothendieck inequality and applications*, Ark. Math. **17** (1979), 51–68.
- [Bo] A. Borzyszkowski, *Unconditional decompositions and local unconditional structures in some subspaces of  $L_p$ ,  $1 \leq p < 2$* , Studia Math. **76** (1983), 267–278.
- [B1] J. Bourgain, *New Banach space properties of the disc algebra and  $H^\infty$* , Acta Math. **152** (1984), 1–48.
- [B2] ———, *Bilinear forms on  $H_\infty$  and bounded bianalytic functions*, Trans. Amer. Math. Soc. **286** (1984), 313–337.
- [B3] ———, *On martingale transforms in finite dimensional lattices with an appendix on the  $K$ -convexity constant*, Math. Nachr. **119** (1984), 41–53.
- [B-D] J. Bourgain and W. J. Davis, *Martingales transforms and complex uniform convexity* (to Trans. Amer. Math. Soc. **294** (1986), 501–515).
- [B-P] J. Bourgain and G. Pisier, *A construction of  $\mathcal{L}_\infty$ -spaces and related Banach spaces*, Bol. Soc. Brasil Mat. **14** (1983), 109–123.
- [Bu] J. Bunce, *The similarity problem for representations of  $C^*$ -algebras*, Proc. Amer. Math. Soc. **81** (1981), 409–414.
- [C1] E. Christensen, *On non self-adjoint representations of  $C^*$ -algebras*, Amer. J. Math. **103** (1981), 817–833.
- [C2] ———, *Extensions of derivations. II*, Math. Scand. **50** (1982), 111–122.
- [C3] ———, *Similarities of  $\Pi_1$  factors with property  $\Gamma$* , København, 1984 (preprint).
- [Co] A. Connes, *On the cohomology of operator algebras*, J. Funct. Anal. **28** (1978), 248–253.
- [D-K] D. Dacunha-Castelle and J. L. Krivine, *Applications des ultraproducts à l'étude des espaces et des algèbres de Banach*, Studia Math. **41** (1972), 315–334.
- [D-J] W. Davis and W. Johnson, *Compact, non nuclear operators*, Studia Math. **51** (1974), 81–85.
- [D-G-T] W. Davis, D. J. H. Garling and N. Tomczak-Jaegermann, *The complex convexity of quasi-normed spaces*, J. Funct. Anal. **55** (1984), 110–150.
- [De] D. Dean, *The equation  $L(E, X^{**}) = L(E, X)^{**}$  and the principle of local reflexivity*, Proc. Amer. Math. Soc. **40** (1973), 146–148.
- [DG] M. Déchamps Gondim, *Analyse harmonique, analyse complexe et géométrie des espaces de Banach [d'après J. Bourgain]*, Séminaire N. Bourbaki, 83/84, Exposé No. 623, Astérisque, Soc. Math. France **121–122** (1985), 171–195.
- [D-U] J. Diestel and J. J. Uhl, *Vector measures*, Math. Surveys, No. 15, Amer. Math. Soc., Providence, R.I., 1977.
- [D-S] S. Dilworth and S. Szarek, *The cotype constant and an almost Euclidean decomposition for finite dimensional normed spaces*, Israel J. Math. **52** (1985), 82–96.
- [D-P-R] E. Dubinsky, A. Pełczyński and H. P. Rosenthal, *On Banach spaces  $X$  for which  $\Pi_2(\mathcal{L}_\infty, X) = B(\mathcal{L}_\infty, X)$* , Studia Math. **44** (1972), 617–648.
- [E] P. Enflo, *A counterexample to the approximation problem in Banach spaces*, Acta. Math. **130** (1973), 309–317.

- [F] T. Figiel, *On the moduli of convexity and smoothness*, Studia Math. **56** (1976), 121–155.
- [F-J] T. Figiel and W. Johnson, *A uniformly convex space which contains no  $l_p$* , Compositio Math. **29** (1974), 179–190.
- [F-J-T] T. Figiel, W. Johnson and L. Tzafriri, *On Banach lattices and spaces having local unconditional structure, with applications to Lorentz function spaces*, J. Approx. Theory **13** (1975), 395–412.
- [F-K-P] T. Figiel, S. Kwapien and A. Pełczyński, *Sharp estimates for the constants of local unconditional structure of Minkowski spaces*, Bull. Acad. Polon. Sci. Ser. Sci. Math. **25** (1977), 1221–1226.
- [F-L-M] T. Figiel, J. Lindenstrauss and V. Milman, *The dimensions of almost spherical sections of convex bodies*, Acta Math. **139** (1977), 53–94.
- [F-T] T. Figiel and N. Tomczak-Jaegermann, *Projections onto hilbertian subspaces of Banach spaces*, Israel J. Math. **33** (1979), 155–171.
- [Fo1] J. Fournier, *On a theorem of Paley and the Littlewood conjecture*, Ark. Math. **17** (1979), 199–216.
- [Fo2] ———, *Multilinear Grothendieck inequalities via the Schur algorithm*, Canad. Math. Soc. Conf. Proc., Vol. 1, Amer. Math. Soc., Providence, R.I., 1981.
- [G-G] D. J. H. Garling and Y. Gordon, *Relations between some constants associated with finite dimensional Banach spaces*, Israel J. Math. **9** (1971), 346–361.
- [Glu1] E. Gluskin, *The diameter of the Minkowski compactum is roughly equal to  $n$* , Funct. Anal. Appl. **15** (1981), 72–73.
- [Glu2] ———, *Finite dimensional analogues of spaces without a basis*, Dokl. Akad. Nauk SSSR **261** (1981), 1046–1050. (Russian)
- [G-L1] Y. Gordon and D. Lewis, *Absolutely summing operators and local unconditional structures*, Acta Math. **133** (1974), 27–48.
- [G-L2] ———, *Banach ideals on Hilbert spaces*, Studia Math. **54** (1975), 161–172.
- [G-R-S] R. Graham, B. Rothschild and J. Spencer, *Ramsey theory*, Wiley, New York, 1980.
- [G1] A. Grothendieck, *Résumé de la théorie métrique des produits tensoriels topologiques*, Bol. Soc. Mat. São-Paulo **8** (1956), 1–79.
- [G2] ———, *Produits tensoriels topologiques et espaces nucléaires*, Mem. Amer. Math. Soc., no. 16, 1955.
- [H1] U. Haagerup, *The Grothendieck inequality for bilinear forms on  $C^*$ -algebras*, Adv. in Math. **56** (1985), 93–116.
- [H2] ———, *Solution of the similarity problem for cyclic representations of  $C^*$ -algebras*, Ann. of Math. (2) **118** (1983), 215–240.
- [H3] ———, *All nuclear  $C^*$ -algebras are amenable*, Invent. Math. **74** (1983), 305–319.
- [H4] ———, *The best constants in the Khintchine inequality*, Studia Math. **70** (1982), 231–283.
- [H5] ———, *An example of a non nuclear  $C^*$ -algebra which has the metric approximation property*, Invent. Math. **50** (1978), 279–293.
- [He] S. Heinrich, *Ultraproducts in Banach space theory*, J. Reine Angew. Math. **313** (1980), 72–104.
- [Ho] K. Hoffman, *Banach spaces of analytic functions*, Prentice-Hall, Englewood Cliffs, N.J., 1962.
- [HJ1] J. Hoffman-Jørgensen, *Sums of independent Banach space valued random variables*, Aarhus Univ. Preprint series 72–73, no. 15, Aarhus.
- [HJ2] ———, *Probability in Banach spaces*, Lecture Notes in Math., vol. 598, Springer-Verlag, Berlin, 1977.
- [HJ-P] J. Hoffmann-Jørgensen and G. Pisier, *The law of large numbers and the central limit theorem in Banach spaces*, Ann. Probab. **4** (1976), 587–599.
- [Ja] R. C. James, *Uniformly non square Banach spaces*, Ann. of Math. (2) **80** (1964), 542–550.
- [Jo] F. John, *Extremum problems with inequalities as subsidiary conditions*, Courant Anniversary Volume. Interscience, New York, 1948, pp. 187–204.
- [J] W. B. Johnson, *On finite dimensional subspaces of Banach spaces with local unconditional structure*, Studia Math. **51** (1974), 225–240.
- [J-L-S] W. Johnson, J. Lindenstrauss and G. Schechtman, *On the relation between several notions of unconditional structure*, Israel J. Math. **37** (1980), 120–129.

- [J-T1] W. Johnson and L. Tzafriri, *On the local structure of subspaces of Banach lattices*, Israel J. Math. **20** (1975), 292–299.
- [J-T2] ———, *Some more Banach spaces which do not have local unconditional structure*, Houston J. Math. **3** (1977), 55–60.
- [K-S] M. Kadec and M. Snobar, *Certain functionals on the Minkowski compactum*, Mat. Zametki. **10** (1971), 453–658. (Russian)
- [Kad] I. Kadison, *On the orthogonalization of operator representations*, Amer. J. Math. **77** (1955), 600–620.
- [Kai-S] S. Kaijser and A. Sinclair, *Projective tensor products of  $C^*$ -algebras*, Math. Scand. **55** (1984), 161–187.
- [Kai] S. Kaijser, *A simple-minded proof of the Pisier-Grothendieck inequality*, Proc. Univ. of Connecticut 80/81, Lecture Notes in Math., vol. 995, Springer-Verlag, pp. 33–35.
- [K-R] N. Kalton and J. Roberts, *A rigid subspace of  $L_0$* , Trans. Amer. Math. Soc. **266** (1981), 645–654.
- [Ka] B. S. Kašin, *Sections of some finite dimensional sets and classes of smooth functions*, Izv. Acad. Nauk. SSSR **41** (1977). (Russian)
- [Ke] T. Ketonen, *On unconditionality in  $L_p$  spaces*, Ann. Acad. Sci. Fenn. Ser. A I Math. Dissertationes **35** (1981).
- [Ki1] S. V. Kisliakov, *On spaces with “small” annihilators*, Zap. Nauchn. Sem. Leningrad. Otdel. Math. Inst. Stekov (LOMI) **65** (1976), 192–195. (Russian)
- [Ki2] ———, *What is needed for 0-absolutely summing to be nuclear?*, Complex Analysis and Spectral Theory, Seminar (Leningrad 1979–80), Lecture Notes in Math., vol. 864, Springer-Verlag, 1981, p. 336.
- [K] H. König, *Spaces with large projection constants* Israel J. Math. **50** (1985), 181–188.
- [K-L-L] H. König, D. Lewis and P. K. Lin, *Finite dimensional projection constants*, Studia Math. **75** (1983), 341–358.
- [Kri1] J. L. Krivine, *Sous-espaces de dimension finie des espaces de Banach réticulés*, Ann. of Math. (2) **104** (1976), 1–29.
- [Kri2] ———, *Théorèmes de factorisation dans les espaces réticulés*, Séminaire Maurey-Schwartz 73–74, Exposé 22, École Polytechnique, Paris.
- [Kri3] ———, *Sur la constante de Grothendieck*, C. R. Acad. Sci. Paris Ser. A **284** (1977), 445–446.
- [Kri4] ———, *Sur la complexification des opérateurs de  $L^\infty$  dans  $L^1$* , C. R. Acad. Sci. Paris Ser. A **284** (1977), 377–379.
- [Kri5] ———, *Constantes de Grothendieck et fonctions de type positif sur les sphères*, Adv. in Math. **31** (1979), 16–30.
- [Kri6] ———, *Sur un théorème de Kasin*, Séminaire d’Analyse Fonctionnelle 83/84, Université Paris 7.
- [Kw1] S. Kwapien, *Isomorphic characterizations of inner product spaces by orthogonal series with vector coefficients*, Studia Math. **44** (1972), 583–595.
- [Kw2] ———, *Isomorphic characterizations of Hilbert spaces by orthogonal series with vector valued coefficients*, Exposé No. 8, Séminaire Maurey-Schwartz 72–73, École Polytechnique, Paris.
- [Kw3] ———, *On operators factorizable through  $L_p$ -space*, Bull. Soc. Math. France Mém. **31–32** (1972), 215–225.
- [Kw4] ———, *A linear topological characterization of inner product spaces*, Studia Math. **38** (1970), 277–278.
- [K-P1] S. Kwapien and A. Pełczyński, *Absolutely summing operators and translation invariant spaces of functions on compact Abelian groups*, Math. Nachr. **94** (1980), 303–340.
- [K-P2] ———, *Remarks on absolutely summing translation invariant operators from the disc algebra and its dual into a Hilbert space*, Michigan Math. J. **25** (1978), 173–181.
- [La] E. C. Lance, *Tensor products and nuclear  $C^*$ -algebras*, Operator Algebras and Applications, Proc. Sympos. Pure Math., Vol. 38, Part 1, Amer. Math. Soc., Providence, R.I. 1980, pp. 379–400.
- [Le] D. Lewis, *Finite dimensional subspaces of  $L_p$* , Studia Math. **63** (1978), 207–212.
- [Li-Pi] W. Linde and A. Pietsch, *Mappings of Gaussian measures of cylindrical sets in Banach spaces* Teor. Veroyatnost. i Primenen. **19** (1974), 472–487. (Russian)

- [L] J. Lindenstrauss, *The geometric theory of the classical Banach spaces*, Actes Congr. Internat. Math.-Nice **2** (1970), 365–372.
- [L-P] J. Lindenstrauss and A. Pełczyński, *Absolutely summing operators in  $L_p$  spaces and their applications*, Studia Math. **29** (1968), 275–326.
- [L-R] J. Lindenstrauss and H. P. Rosenthal, *The  $\mathcal{L}_p$  spaces*, Israel J. Math. **7** (1969), 325–349.
- [L-T1] M. Lindenstrauss and L. Tzafriri, *Classical Banach spaces*. I, Springer-Verlag, Berlin and New York, 1977.
- [L-T2] ———, *Classical Banach spaces*. II: *Function spaces*, Springer-Verlag, Berlin and New York, 1979.
- [L-Z] J. Lindenstrauss and M. Zippin, *Banach spaces with sufficiently many Boolean algebras of projections*, J. Math. Anal. and Appl. **25** (1969), 309–320.
- [Lo-R] J. Lopez and K. Ross, *Sidon sets*, Dekker, New York, 1975.
- [Ma-Pi] M. B. Marcus and G. Pisier, *Random Fourier series with applications to harmonic analysis*, Ann. of Math. Stud., no. 101, Princeton Univ. Press, Princeton, N.J., 1981.
- [M1] B. Maurey, *Théorèmes de factorisation pour les opérateurs linéaires à valeurs dans un espace  $L^p$* , Astérisque **11** (1974).
- [M2] ———, *Espaces de cotype  $p$* , Séminaire Maurey-Schwartz 72/73, Exposé no. 7, École Polytechnique, Paris.
- [M3] ———, *Nouvelle démonstration d'un théorème de Grothendieck*, Séminaire Maurey-Schwartz 72/73, Exposé no. 22, École Polytechnique, Paris.
- [M4] ———, *Quelques problèmes de factorisation d'opérateurs linéaires*, Actes Congr. Internat. Math. Vancouver 2 (1974), 75–79.
- [M5] ———, *Type et cotype dans les espaces munis de structure locale inconditionnelle*, Séminaire Maurey-Schwartz 73/74, Exposé 24–25, École Polytechnique, Paris.
- [M6] ———, *Un théorème de prolongement*, C. R. Acad. Sci. Paris A **279** (1974), 329–332.
- [M-P] B. Maurey and G. Pisier, *Séries de variables aléatoires vectorielles indépendantes et propriétés géométriques des espaces de Banach*, Studia Math. **58** (1976), 45–90.
- [Me] J. F. Méla, *Mesures  $\varepsilon$ -itempotentes de norme bornée*, Studia Math. **72** (1982), 131–149.
- [Mi] V. Milman, *Almost Euclidean quotient spaces of subspaces of a finite-dimensional normed space*, preprint, 1984, Proc. Amer. Math. Soc. **94** (1985), 445–449.
- [Mi-Sha] V. Milman and M. Sharir, *A new proof of the Maurey-Pisier theorem*, Israel J. Math. **33** (1979), 73–87.
- [Mi-Sche] V. Milman and G. Schechtman, *Asymptotic theory of finite dimensional normed spaces*, Springer Lecture Notes. vol. **1200**, Springer-Verlag, 1986.
- [Mi-W] V. Milman and H. Wolfson, *Minkowski spaces with extremal distance from the Euclidean space*, Israel J. Math. **29** (1978), 113–130.
- [N] L. Nachbin, *A theorem of Hahn-Banach type for linear transformations*, Trans. Amer. Math. Soc. **68** (1950), 28–46.
- [Pa] V. Paulsen, *Completely bounded maps on  $C^*$ -algebras and invariant operator ranges*, Proc. Amer. Math. Soc. **86** (1982), 91–96.
- [Ped] G. Pedersen,  *$C^*$ -algebras and their automorphism groups*, Academic Press, London, 1979.
- [Pe1] A. Pełczyński, *Banach spaces of analytic functions and absolutely summing operators*, CBMS Regional Conf. Ser. in Math., Vol. 30, Amer. Math. Soc., Providence, R.I. 1977.
- [Pe2] ———, *Sur certaines propriétés isomorphiques nouvelles des espaces de Banach de fonctions holomorphes  $A$  et  $H^\infty$* , C. R. Acad. Sci. Paris A **279** (1974), 9–12.
- [Pe3] ———, *A characterization of Hilbert Schmidt operators*, Studia Math. **28** (1967), 355–360.
- [Pe4] ———, *Geometry of finite dimensional spaces and operator ideals*, Notes in Banach spaces (E. Lacey, ed.), Univ. of Texas Press, 1980, pp. 81–181.
- [P1] A. Pietsch, *Absolut  $p$ . summierende Abbildungen in normierten Räumen*, Studia Math. **28** (1967), 333–353.
- [P2] ———, *Operator ideals*, North-Holland, Berlin, 1978.
- [Pi1] G. Pisier, *Holomorphic semi-groups and the geometry of Banach spaces*, Ann. of Math. (2) **115** (1982), 375–392.
- [Pi2] ———, *On the duality between type and cotype*, Martingale Theory in Harmonic Analysis and Banach Spaces (Proc., Cleveland, 1981), Lecture Notes in Math., vol. 939, Springer-Verlag.

- [Pi3] ———, *K-convexity* (Proc. Res. Workshop on Banach Space Theory, June 29–July 31, 1981), Univ. of Iowa Press, 1982.
- [Pi4] ———, *Quotients of Banach spaces of cotype  $q$* , Proc. Amer. Math. Soc. **85** (1982), 32–36.
- [Pi5] ———, *On the dimension of the  $l_p^n$  subspaces of Banach spaces, for  $1 \leq p < 2$* , Trans. Amer. Math. Soc. **276** (1983), 201–211.
- [Pi6] ———, *Some results on Banach spaces without local unconditional structure*, Compositio Math. **37** (1978), 3–19.
- [Pi7] ———, *Grothendieck's theorem for non-commutative  $C^*$ -algebras with an appendix on Grothendieck's constants*, J. Funct. Anal. **29** (1978), 397–415.
- [Pi8] ———, *Une nouvelle classe d'espaces vérifiant le théorème de Grothendieck*, Ann. Inst. Fourier (Grenoble) **28** (1978), 69–90.
- [Pi9] ———, *Un théorème sur les opérateurs entre espaces de Banach qui se factorisent par un espace de Hilbert*, Ann. École Norm. Sup. **13** (1980), 23–43.
- [Pi10] ———, *Counterexamples to a conjecture of Grothendieck*, Acta Math. **151** (1983), 181–208.
- [Pi11] ———, *Remarques sur un résultat non publié de B. Maurey*, Séminaire d'Analyse Fonctionnelle. Exposé no. 5, École Polytechnique, Palaiseau.
- [Pi12] ———, *Les inégalités de Khintchine-Kahane, d'après C. Borell*, Séminaire sur la géométrie des Espaces de Banach (1977–78), Exp. No. 7, 14 pp., École Polytech., Palaiseau, 1978.
- [Re] S. Reisner, *On Banach spaces having the property  $G.L.$* , Pacific J. Math. **83** (1979), 505–521.
- [Ri1] J. R. Ringrose, *Operator algebras and their Abelian subalgebras*, Proc. Internat. Congress Math., Vancouver 2 (1974), 105–110.
- [Ri2] ———, *Linear mappings between operator algebras*, Symposia Math., vol. 20, Academic Press, New York, 1976.
- [R1] H. P. Rosenthal, *A characterization of Banach spaces containing  $l_1$* , Proc. Nat. Acad. Sci. U.S.A. **71** (1974), 2411–2413.
- [R2] ———, *On subspaces of  $L^p$* , Ann. of Math. (2) **97** (1973), 344–373.
- [R3] ———, *A characterization of  $c_0$  and some remarks concerning the Grothendieck property*, Longhorn Notes, Univ. of Texas, Functional Analysis Seminar 82/83, pp. 95–108.
- [Ru1] W. Rudin, *Fourier analysis on groups*, Interscience Tracts in Pure and Applied Mathematics, No. 12, Interscience, New York, 1962.
- [Ru2] ———, *Trigonometric series with gaps*, J. Math. Mech. **9** (1960), 203–228.
- [Sa] S. Sakai, *Developments in the theory of derivations in  $C^*$ -algebras*, Proc. Internat. Conf. Operator Algebras (Leipzig, 1977), Teubner, Leipzig, 1978, pp. 234–240.
- [Sin] A. Sinclair, *Automatic continuity of linear operators*, London Math. Soc. Lectures Notes, no. 21, Cambridge Univ. Press, 1976.
- [Sc] C. Schütt, *Unconditionality in tensor products*, Israel J. Math. **31** (1978), 209–216.
- [S] L. Schwartz, *Geometry and probability in Banach spaces*, Lecture Notes in Math., vol. 852, Springer-Verlag, 1981.
- [Sh] S. Shelah, *A Banach space with few operators*, Israel J. Math. **30** (1978), 181–191.
- [Si] S. Simons, *Local reflexivity and  $(p, q)$ -summing maps*, Math. Ann. **198** (1972), 335–344.
- [Sza] A. Szankowski,  *$B(H)$  does not have the approximation property*, Acta Math. **147** (1981), 89–108.
- [Sz1] S. Szarek, *The finite dimensional basis problem with an appendix on nets of Grassmann manifold*, Acta Math. **151** (1983), 153–180.
- [Sz2] ———, *On Kasin's euclidean orthogonal decomposition of  $l_1^n$* , Bull. Acad. Polon. Sci. **26** (1978), 691–694.
- [Sz3] ———, *Volume estimates and nearly Euclidean decompositions of normed spaces*, Séminaire d'Analyse Fonctionnelle, (1979–80), Exp. No. 25, École Polytechnique, Palaiseau.
- [Sz4] ———, *On the best constants in the Khintchine inequality*, Studia Math. **58** (1978), 197–208.
- [Sz5] ———, *A note on: "Unconditionality in tensor products" by C. Schütt*, Colloq. Math. **45** (1981), 273–276.
- [S-T] S. Szarek and N. Tomczak-Jaegermann, *On nearly Euclidean decomposition for some classes of Banach spaces*, Compositio Math. **40** (1980), 367–385.



- [TJ] N. Tomczak-Jaegermann, *The moduli of smoothness and convexity and the Rademacher averages of the trace classes  $S_p$*  ( $1 \leq p < \infty$ ), *Studia Math.* **50** (1974), 163–182.
- [To1] A. Tonge, *Polarisation and the complex Grothendieck inequality*, *Math. Proc. Cambridge Philos. Soc.* **95** (1984), 313–318.
- [To2] ———, *Banach algebras and absolutely summing operators*, *Math. Proc. Cambridge Philos. Soc.* **80** (1976), 465–473.
- [Tz] L. Tzafriri, *On Banach spaces with unconditional basis*, *Israel J. Math.* **17** (1974), 84–93.
- [V1] N. Th. Varopoulos, *Une remarque sur les ensembles de Helson*, *Duke Math. J.* **43** (1976), 387–390. Also see Séminaire Maurey-Schwartz 1976–77, Exposé no. 12.
- [V2] ———, *A theorem on operator algebras*, *Math. Scand.* **37** (1975), 173–182.
- [W] G. Wittstock, *Ein operatorwertiger Hahn-Banach Satz*, *J. Funct. Anal.* **40** (1981), 127–150.

