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Dedicated to the memory of
Ron DiPerna

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Contents

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Preface

The purpose of this monograph is to present a survey of and an introduction to the central aspects of nonlinear wave equations in the absence of shocks. The theory began in the 1960s with the paper of K. Jörgens and the stimulating suggestions of I. Segal. In spite of a great deal of recent activity, some major questions remain open, such as: (1) sharp conditions for the global existence of solutions with arbitrary initial data and (2) the global phase portrait in the presence of periodic solutions and traveling waves.

I have attempted to state the sharpest results to date, emphasizing those results which appear to be definitive and de-emphasizing those which appear to be technicalities. Most of the proofs are not complete but only the main ideas are presented and the reader is invited to pursue the references.

In order to keep these lectures to a moderate length, some aspects of nonlinear wave equations have been completely omitted, while others have been only briefly mentioned. These choices are based on my own personal predilections. Some *omitted* topics are the following. (1) Local existence; i.e., solutions which may exist for only a finite time: see Kato [**Ka86**]. (2) Boundaries and variable coefficients. We are concerned almost exclusively with equations in free space $\mathbf{R}^n$ which are translation-invariant. Thus, for instance, reflection and refraction are not considered. (3) Shock waves: see Lax [**L**]. (4) Solitons: see Newell [**N**]. (5) Free vibrations; that is, solutions periodic in time, known to exist for NLW in one dimension in a bounded interval. The basic result is due to Rabinowitz; see Brezis [**Br**] for a survey. (6) Sets of attraction. In the presence of dissipation these have been proved to be finite dimensional; see Hale [**Ha**] or Temam [**Te**].

Equation (B.A) will refer to equation (A) in Chapter B; similarly for Theorem B.A. Equation (A) or Theorem A will refer to the current chapter.

I want to thank the many friends who have been my collaborators over the years. I specificially thank those who have made suggestions and corrections to the manuscript, including Takis Souganidis, Bob Glassey, Piotr Chrusciel and Yoshio Tsutsumi. Jeng-Eng Lin deserves special thanks for originally suggesting the CBMS conference and for cheerfully seeing to all the details

and making the practical arrangements to assure its success. Thanks are also due to the Department of Mathematical Sciences of George Mason University for its hospitality. Finally, I thank Dale Cavanaugh for her excellent typing of the manuscript.

Walter Strauss
Brown University
April 1, 1989

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(Continued in the back of this publication)

The theory of nonlinear wave equations in the absence of shocks began in the 1960s. Despite a great deal of recent activity in this area, some major issues remain unsolved, such as sharp conditions for the global existence of solutions with arbitrary initial data, and the global phase portrait in the presence of periodic solutions and traveling waves.

This book, based on lectures presented by the author at George Mason University in January 1989, seeks to present the sharpest results to date in this area. The author surveys the fundamental qualitative properties of the solutions of nonlinear wave equations in the absence of boundaries and shocks. These properties include the existence and regularity of global solutions, strong and weak singularities, asymptotic properties, scattering theory and stability of solitary waves. Wave equations of hyperbolic, Schrödinger, and KdV type are discussed, as well as the Yang–Mills and the Vlasov–Maxwell equations.

The book offers readers a broad overview of the field and an understanding of the most recent developments, as well as the status of some important unsolved problems. Intended for mathematicians and physicists interested in nonlinear waves, this book would be suitable as the basis for an advanced graduate-level course.

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