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Introduction

An NSF-CBMS regional conference on "K-theory and Dynamics" was held in Gainesville, Florida from January 9th to the 14th, 1989. The conference was funded by the National Science Foundation and hosted by the University of Florida. C. W. Stark was the conference director and the organizing committee consisted of D. Fried, L. E. Jones, and J. B. Wagoner. F. T. Farrell was the principal lecturer and this book is based on those lectures. The more technical topics have been deleted. Some results which were obtained since the conference are discussed in an epilogue (Chapter 6). This epilogue also contains some theorems, not mentioned at the conference but obtained in the last ten years, showing there are many aspherical manifolds which are not classical. The ten lectures were primarily concerned with classical aspherical manifolds; e.g., those arising as double coset spaces of Lie groups or from synthetic geometry. The main problem addressed was the topological characterization of compact (closed) classical aspherical manifolds. The problem has been mostly solved; 3-dimensional and 4-dimensional manifolds present the most important unsolved aspects. (Poincaré's conjecture is closely related to the 3-dimensional problem.)

We wish to express our special thanks to Chris Stark. His efforts made possible both the conference and this book.

F. Thomas Farrell
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6.3. Corollary (Farrell and Jones [**1990**]). *Let* A *be any finitely generated free abelian group, including* 0. *Then* $\mathrm{Wh}(\pi_1 M \times A) = 0$. *Consequently,* $\mathrm{Wh}\,\pi_1 M$, $\widetilde{K}_0(\mathbb{Z}\pi_1 M)$, $\mathrm{Nil}(\mathbb{Z}\pi_1 N)$ *and* $K_n(\mathbb{Z}\pi_1 M)$ (*where* $n < 0$) *all vanish.*

Proof. Let T^n be a flat torus such that $\pi_1 T^n \simeq A$. Then $M \times T^n$ is a closed nonpositively curved manifold and $\pi_1 M \times T^n \simeq \pi_1 M \times A$. We may assume $n > 4$. Thus it suffices to consider the case where $A = 0$ and $\dim M > 4$. Let $x \in \mathrm{Wh}\,\pi_1 M$ and (W, M) be a h-cobordism such that $\tau(W, M) = x$. Let N be the top of W and $f\colon N \to M$ be the composite of the inclusion N into W with a retraction of W onto M, then f is homotopic to a homeomorphism $\varphi\colon N \to M$, because of 5.1. Let $\mathscr{M}$ be the mapping torus of φ. There is a homotopy equivalence $g\colon \mathscr{M} \to M \times S^1$ such that $\tau(g)$ is $\sigma_\#(x)$, where $\sigma\colon \pi_1 M \to \pi_1 M \times C$ is the inclusion map onto the first factor. Note that $\sigma_\#$ is monic, because Wh is a functor. Consequently it suffices to show that $\tau(g) = 0$. But g is homotopic to a homeomorphism because of 5.1. Therefore, 1.13 shows that $\tau(g) = 0$. □

Farrell and Jones [**1990**] show that the weak homotopy type of $\mathscr{P}(M)$ is calculable in terms of $\mathscr{P}(S^1)$ through a stable range of dimensions. This generalizes 3.14. The calculation is in terms of the structure of families of closed geodesics in M. When M is a locally symmetric space of rank > 1, this structure can be complicated. Here are some consequences of this calculation and of a generalization of 6.1 which is analogous to 5.8.

6.4. Theorem (Farrell and Jones [**1990**]). *Let* n *be any positive integer and* $\nu = [(n+1)/2]!$, *then* $\mathrm{Wh}_n(\pi_1 M) \otimes \mathbb{Z}[\frac{1}{\nu}] = 0$ *and consequently*

$$K_n(\mathbb{Z}\pi_1 M) \otimes \mathbb{Q} = H_n(M, \mathbb{Q}) \oplus (\oplus_{i=1}^{\infty} H_{n-1-4i}(M, \mathbb{Q})).$$

Assume $m = \dim M$ *is greater than* 10, *then the following calculations hold provided* $1 < n < (m-7)/3$:

$$\pi_n(\mathrm{Top}\,M) \otimes \mathbb{Z}[\tfrac{1}{\nu}] = 0, \quad \textit{and}$$
$$\pi_n(\mathrm{Diff}\,M) \otimes \mathbb{Q} = \begin{cases} \oplus_{j=1}^{\infty} H_{(s+1)-4j}(M, \mathbb{Q}), & \textit{if } m \textit{ is odd} \\ 0, & \textit{if } m \textit{ is even.} \end{cases}$$

We now turn to a discussion of some recent constructions of non-classical closed aspherical manifolds. It had been conjectured that the total space of the universal cover of a closed aspherical manifold must be homeomorphic to Euclidean space. All the classical examples mentioned in Chapter 1 satisfy this conjecture. But Davis [**1983**] constructed, for each dimension $n \geq 4$, n-dimensional closed aspherical manifolds N such that the total space of the universal cover of N is not homeomorphic to $\mathbb{R}^n$.

It also seemed reasonable that a closed aspherical manifold should support a smooth structure. But Davis and Hausmann [**1989**] give examples of closed aspherical manifolds which do not support a smooth structure. In

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fact, Davis and Januszkiewicz **[to appear]** have constructed such examples in every dimension ≥ 4. (Such examples cannot exist in dimensions 1, 2 and 3.) Moreover, they construct a four-dimensional closed aspherical manifold which cannot be triangulated; i.e., it is not homeomorphic to the geometric realization of a finite simplicial complex.

It had also been conjectured that every strictly negatively curved, closed Riemannian manifold is diffeomorphic to a locally symmetric space. Mostow and Siu **[1980]** gave a four-dimensional counterexample to this conjecture. Gromov and Thurston **[1987]** gave counterexamples in every dimension > 3.

Let N_1 and N_2 be a pair of closed strictly negatively curved manifolds with isomorphic fundamental groups. By results of Eells and Sampson **[1964]**, Hartman **[1967]** and Al'ber **[1968]**, there exists a unique harmonic map f: $N_1 \to N_2$ inducing this isomorphism. Since N_1 and N_2 are aspherical, f is a homotopy equivalence. When N_1 and N_2 are hyperbolic (and $\dim N_1 > 2$) 1.25 implies that f is an isometry. This led Lawson and Yau to conjecture that f is always a diffeomorphism. Farrell and Jones **[1989c]** gave the first counterexample to this conjecture. It is a consequence of the following result.

6.5. THEOREM (Farrell and Jones **[1989c]**). *Let N be a closed hyperbolic manifold with $\dim N > 4$. Given any $\epsilon > 0$, there exists a finite sheeted cover $\mathcal{N}$ of N such that the following is true.*

(1) *The connected sum $\mathcal{N}\#\Sigma$, where Σ is any closed smooth manifold homotopically equivalent to S^m, supports a Riemannian metric such that all its sectional curvatures are pinched within ϵ of -1.*

(2) *Let Σ_1 and Σ_2 be any nondiffeomorphic pair of closed smooth manifolds which are both homotopically equivalent to S^m. Then $\mathcal{N}\#\Sigma_1$ is not diffeomorphic to $\mathcal{N}\#\Sigma_2$.*

COUNTEREXAMPLE TO LAWSON–YAU CONJECTURE. Pick a closed smooth manifold Σ such that Σ is homeomorphic to S^m $(m > 4)$ but not diffeomorphic to S^m. Milnor **[1956]** and Kervaire–Milnor **[1963]** give many such Σ. Borel's Theorem 4.5 yields the existence a closed hyperbolic m-dimensional manifold N^m. Let ϵ be any positive number less than 1 and $\mathcal{N}$ be the covering space of N posited in 6.5. Then $N_1 = \mathcal{N}$ and $N_2 = \mathcal{N}\#\Sigma$, where $\mathcal{N}$ is given the hyperbolic metric induced from N and $\mathcal{N}\#\Sigma$ the metric posited in (1) of 6.5.

REMARK. A partial positive result related to the Lawson–Yau conjecture is a consequence of 6.1 (cf. Farrell and Jones **[1989 b]**). Namely, there is always a homeomorphism inducing the isomorphism between $\pi_1 N_1$ and $\pi_1 N_2$ provided $\dim N_1 \neq 3, 4$.

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