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number 76

AMERICAN MATHEMATICAL SOCIETY · FOUNDED 1888 · ΑΓΕΩΜΕΤΡΗΤΟΣ ΜΗΔΕΙΣ ΕΙΣΙΤΩ

Klaus Schmidt

ALGEBRAIC IDEAS IN ERGODIC THEORY

NATIONAL · SCIENCE · FOUNDATION

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published by the american mathematical society

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http://dx.doi.org/10.1090/cbms/076

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Published for the
Conference Board of the Mathematical Sciences
by the
American Mathematical Society
Providence, Rhode Island

Expository Lectures
from the CBMS Regional Conference
held at the University of Washington, Seattle
July 1989

Research supported by National Science Foundation Grants DMS-8814159 and DMS-8820716, as well as additional grants from the IBM Watson Research Center, the IBM Almaden Research Center, and the University of Washington.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 28D05, 28D15, 28D20, 54C70.

Library of Congress Cataloging-in-Publication Data

Schmidt, Klaus, 1943-

Algebraic ideas in ergodic theory/Klaus Schmidt.

p. cm.—(Regional conference series in mathematics, ISSN 0160-7642; v. 76)

"A series of ten lectures at the Conference Board of the Mathematical Sciences Regional Conference, Seattle, Washington, July 1989."

Includes bibliographical references and index.

ISBN 0-8218-0727-7

1. Ergodic theory—Congresses. 2. Operator algebras—Congresses. 3. Markov processes—Congresses. I. Conference Board of the Mathematical Sciences. II. Title.

QA611.7.S36 1990 90-877
515.42—dc20 CIP

The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability. ♾
This publication was typeset using $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX,
the American Mathematical Society's TeX macro system.

10 9 8 7 6 5 4 3 2 1 95 94 93 92 91 90

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Notation

$\alpha^{\mathscr{N}}$	$\mathbb{Z}^d$-action defined by a module $\mathscr{N}$	68
$\mathscr{A}(\mathbf{R}), \mathscr{A}'(\mathbf{R})$	von Neumann algebras of a nonsingular equivalence relation	11
$B^1(\mathbf{R}, A)$	set of coboundaries of $\mathbf{R}$ with coefficients in a group A	18
$\mathbf{B}(H)$	= the set of bounded linear operators on a Hilbert space H	10
β_P	beta-function of a Markov shift	23
$\mathbb{C}$	= the complex numbers	
c_B	restriction of a cocycle c to a set B	21
c_P	generator of Γ_P/Δ_P	23
$\mathscr{C}(\mathbf{R})$	a C^*-algebra of an equivalence relation $\mathbf{R}$	14
$\delta_{a,b}$	= the Kronecker delta ($\delta_{a,b} = 1$ if $a = b$, and $\delta_{a,b} = 0$ otherwise)	11
Δ_P	a group associated with a nonnegative matrix P	22
$(\mathscr{D}(\mathbf{R}), \mathscr{D}(\mathbf{R})^+)$	dimension module	48
$\mathscr{E}(c), \mathscr{E}^-(c)$	essential range of a cocycle c	19
$\mathrm{GL}(n, \mathbb{Z})$	= the group of invertible $n \times n$ matrices with integer entries	40, 72
Γ_P	a group associated with a nonnegative matrix P	22
$h(\cdot)$	entropy	7, 56
$H^1(\mathbf{R}, A)$	first cohomology group of $\mathbf{R}$ with coefficients in an abelian group A	18 24
$\mathscr{I}_{\mathbf{R}:S}$	index cocycle of a subrelation	
k_π	= algebraic closure of the prime field $F_\pi = \mathbb{Z}_\pi$ of characteristic π	45
$K_0(\mathscr{A})$	dimension group of a C^*-algebra	
L_V	nonsingular automorphism of $\mathbf{R}$ defined by an automorphism $V \in [\mathbf{R}]$	13
L_V, L'_V	operators in $\mathscr{A}(R)$ and $\mathscr{A}(R)$ associated with an automorphism $V \in [R]$	11
$\lambda(V)$	module of $V \in \mathscr{N}(R)$	32
μ_B	= restriction of a measure μ to a measurable set B with $\mu(B) > 0$	
m_P	measure of maximal entropy on a Markov shift	7, 8
μ_P	Markov measure on a two-sided Markov shift X_P	6
$\mu_{\mathbf{R}}, \mu_{\mathbf{R}}^{(L)}, \mu_{\mathbf{R}}^{(R)}$	measures on a nonsingular equivalence relation $\mathbf{R}$	5
$\mathscr{M}(\mathbf{R}), \mathscr{M}'(\mathbf{R})$	algebra of multiplication operators in $\mathscr{A}'(\mathbf{R})$ and $\mathscr{A}(\mathbf{R})$	11
$\mathbb{N}$	$= \{0, 1, 2, \dots\}$	
$\mathbb{N}^\times$	$= \{1, 2, 3, \dots\}$	

ν_P	Markov measure on a one-sided Markov shift Y_P	8
$\mathscr{N}(\mathbf{R})$	normalizer of an equivalence relation $\mathbf{R}$	32
P, $\mathbf{P}$	nonnegative, irreducible matrix and the associated stochastic matrix	6
$\pi(V)$	outer period of $V \in \mathscr{N}(R)$	32
$\mathbb{Q}$	= the rationals	32
$\mathbb{R}$	= the real numbers	31
$\mathbb{R}_+$	$[0, \infty) \subset \mathbb{R}$	32
$\mathbb{R}^\times$	$= \mathbb{R}\setminus\{0\}$	31
$\mathbb{R}_+^\times$	$= \mathbb{R}_+ \cap \mathbb{R}^\times$	32
$\mathfrak{R}(\alpha)$	= real part of a complex number α	
$[\mathbf{R}]$	full group of an equivalence relation	5
$[[\mathbf{R}]]$	ample group	
$\mathbf{R}_B$	equivalence relation induced on B	5
$r(c)$	cohomology invariant	19
$\mathbf{R}^{(c)}$	skew product relation defined by a cocycle c	18
$\mathbf{R}^{(c,1)}$, $\mathbf{R}^{(c,B)}$	subrelation defined by a cocycle	24
$\mathscr{R}_d$	ring of Laurent polynomials	49
$\mathbf{R}^P$, $\mathbf{R}'^P$	nonsingular equivalence relations on Markov shifts	6
$[\mathbf{R} : \mathbf{S}]$	index of a subrelation	24
$\mathbf{R}^T$	equivalence relation of a nonsingular group action T	5
$\mathbf{R}^V$	equivalence relation of a nonsingular automorphism or endomorphism V)	6, 9
$\mathbf{R}_V$	nonsingular automorphism of $\mathbf{R}$ associated with an automorphism $V \in [\mathbf{R}]$	15
$\rho_{\mathbf{R},\mu}$	Radon–Nikodym derivative of a nonsingular relation R	5
$\lvert S\rvert$	= cardinality of a set S	5
$\mathbb{S}^1$	$= \{z \in \mathbb{C}: \lvert z\rvert = 1\}$	5
$\mathscr{S}_B$	= a σ-algebra $\mathscr{S}$ induced on a set B	
$\sigma^{(F,P)}$	higher dimensional Markov shift	56
$\mathrm{SL}(n, \mathbb{Z})$	= the group of $n \times n$ matrices with integer entries and determinant 1	
$\mathbf{S}^P$, $\mathbf{S}'^P$	nonsingular equivalence relations on Markov shifts	6, 7
σ_P	Markov shift	6, 7
$\mathscr{S}^T$	$= \sigma$-algebra of T-invariant subsets in $\mathscr{S}$, where T is a group action	
$S(f)$	the support of a polynomial f	69, 77
$\mathbf{S}^V$	equivalence relation of a nonsingular endomorphism V	9
$\mathbb{T}$	$= \mathbb{R}/\mathbb{Z}$	
$T^{(c)}$	group action on a skew product	18
T^f	skew product action	28
$\mathscr{T}(c)$	$\mathscr{T}$-set of a cocycle	20
$W^s(x)$, $W^u(x)$	stable and unstable sets of a point x in a two-sided Markov shift	7
$X_{(F,P)}$	higher dimensional Markov shift space	57
$(X^{\mathscr{N}}, \alpha^{\mathscr{N}})$	dynamical system associated with a module	67
X_P	two-sided Markov shift space associated with a nonnegative matrix P	6
X_{P^*}	set of doubly transitive points in X_P	26

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ISBN 978-0-8218-0727-9
9 780821 807279
CBMS/76
ISBN 0-8218-0727-7