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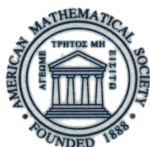
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Regional Conference Series in Mathematics

Number 77

Lectures on Singular Integral Operators

Michael Christ



American Mathematical Society
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National Science Foundation



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SINGULAR INTEGRAL OPERATORS**

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SINGULAR INTEGRAL OPERATORS**

Michael Christ

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to Carol

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Preface

These notes represent an expanded account of lectures delivered at the NSF–CBMS conference on singular integral operators, held at the University of Montana from August 28 through September 1, 1989. They are concerned principally with developments in the subject related to the Cauchy integral on Lipschitz curves and the $T(1)$ theorem. The emphasis is on real-variable techniques, with a discussion of analytic capacity in one complex variable included as an application.

Little of the material here is new, though a few of the proofs seem not to be in the literature. Rather, we have sought to synthesize and expose a body of results and techniques.

Much of the text is introductory in character and is intended to be accessible to the nonexpert, but it is hoped that a variety of readers will find something of interest. Proofs are sometimes incomplete or imprecise; to fill in the missing details would often be a useful exercise for the dedicated student.

Certain aspects of the subject which have been in a state of rapid development during the writing of these notes are not covered, but we have sought to provide a useful introduction to them. One should see the expository work of David [D8], the paper of David and Semmes [DS2], and the article of Jones [JP3]. For a more authoritative and complete treatment of some of the contents of these notes, and much additional material, the reader is urged to consult the books of Meyer [M4].

Thanks to Carol for help in preparing the original lectures, and to Christopher Bishop, Guy David, John Garnett and Peter Jones for helpful advice and comments. Thanks to Rodolfo Torres, Javier Soria, and the other participants in the conference for pointing out errors, typographical and otherwise, in an earlier draft of the notes. Thanks also to Joseph Cima and William Derrick for their outstanding efforts in organizing the conference, and to Glen Munnik for typing a portion of the manuscript. Financial and/or logistical support from the National Science Foundation, Alfred P. Sloan Foundation, and Institut des Hautes Etudes Scientifiques is acknowledged with gratitude.

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