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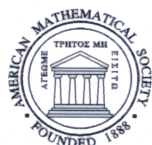
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Number 81

Geometric Analysis and Function Spaces

Steven G. Krantz



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To see a world in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
And eternity in an hour.

—Wm. Blake

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Preface

This short book is not a mathematics monograph in the usual sense of the word. It is more of a manifesto. Its purpose is to illustrate, by way of a number of examples, an interaction between geometry and analysis. In some of these examples the geometry is of the classical Euclidean variety; in others it is Riemannian or Kähler or Finsler geometry. By the same token, some of the analysis that occurs is at the textbook level while other instances use many of the powerhouse tools from the last fifteen or twenty years. The interest is in the symbiosis.

Some of the material in this book is standard. Other parts are well known but have not been recorded. Still others are rather speculative in nature. We shall try to be specific about the places where the picture is incomplete.

In virtually every example that I treat, complex function theory (of either one or several variables) acts as a catalyst. In some instances, the function theory helps to set up or motivate an attractive problem and then quickly fades into the background. In other instances the complex analysis remains nearly to the end, being stripped away in layers to reveal geometry, analysis, and operator theory hidden in its folds.

Another unifying theme of this work will be partial differential equations. In many of the examples, either the Cauchy Riemann equations (of one or several variables), the classical Laplacian, or the Laplace-Beltrami operator for some Riemannian metric will play a role. Partial differential equations will not only serve us as a technical tool, but also as a driving force behind certain questions.

And one question that I want to ask repeatedly is this: “Where do the function spaces that we use come from? Are they still well-suited for the problems that we study today?” Function spaces are a relatively modern invention. The L^p , Hölder/Lipschitz, and Sobolev spaces have all been developed primarily in the twentieth century. These spaces behave canonically under the groups that act on Euclidean space: the translations, rotations, and dilations. And so they should, because they were developed in significant part to study the regularity properties of the classical Laplacian.

From the vantage point of the 1990’s, it is safe to say that the theory of translation invariant operators on Euclidean space, and their mapping properties on the classical function spaces mentioned above, is well understood. Both in the setting of (harmonic) analysis on Euclidean space and of the function theory of several complex variables we have now turned our attention to the analysis of domains. It has become clear in the last fifteen years that the function theory of several complex variables consists in understanding how the Levi geometry of the boundary of a domain influences the holomorphic function theory on the interior. I want to suggest in this context, among others, that it is useful to set aside the classical function spaces and think in terms of new spaces *that are adapted to the geometry of the given situation*.

We shall not throw out the proverbial baby with the bath water: the new function spaces are closely linked to, and motivated by, the classical ones. In many familiar settings the new function spaces are in fact classical ones in disguise. Because we give them new descriptions, and because they are developed canonically for the given context, they will be easier and more natural to treat.

Already in the preface we have introduced a number of themes: analysis, geometry, partial differential equations, and function spaces. These will be present, woven into what we hope is a comfortable tapestry, throughout this book. Of course there is a simple explanation for this setup. It is natural for me to speak here of topics in which I have had a hand in the last twenty years. For synergistic reasons, if for none other, there are bound to be at least tenuous links among these topics. This is thus an opportunity for me to make the links more explicit and, I hope, to clarify some connections and to point in new directions.

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—Steven G. Krantz

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